Kelly Betting

Q: What fraction f^* of our bankroll should we allocate to a bet with a probability p of winning $\$W$ and probability $1-p$ of losing $\$L$?

After n such bets, our portfolio value V_n is given by:

$$V_n = V_0 \underbrace{(1+fW)^{pn}}_{pn \text{ wins}} \underbrace{(1-fL)^{(1-p)n}}_{\text{and } (1-p)n \text{ losses on average}}$$

We choose f to maximise the log return:

$$\ln V_n = \ln V_0 + pn \ln(1+fW) + (1-p)n \ln(1-fL)$$

$$\frac{\partial \ln V_n}{\partial f} = 0 \Rightarrow \frac{pnW}{1+fW} + \frac{(1-p)n(-L)}{1-fL} = 0$$

$$\therefore \frac{pW}{1+fW} = \frac{(1-p)L}{1-fL} \Rightarrow fW(1-p)L + fLpW = pW - (1-p)L$$

$$\Rightarrow f^* = \frac{pW - (1-p)L}{WL}$$