Yale Econ 159 2007 Midterm

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Question 1 (15 marks) 15/15

a) True. By definition, if a strategy s is strictly dominated, then for any possible opponents' choice, there is another strategy s' with strictly better payoff. Hence s cannot be a BR to any opponents' choice because s' is better. 5/5

b) True. Suffices to show that (i) no other candidate's BR to the current situation is to run for election, and (ii) it is not a BR for current candidates to drop out.

_ _ _ $\underline{A}$ _ $\underline{B}$ $\underline{C}$ _ _

i) Because A and B are near the centre, any new entrant C cannot win the election by being further right or left, but they incur the cost of running. Hence it is not a BR for C to run at an extreme position. A and B are not sufficiently extreme that C can capture the middle.

ii) Currently, B has a 50% chance of winning, so has expected payoff $0.5v$. Hence if $v > 2c$, B's BR is to stay in the election. Likewise for A by symmetry

5/5

c) $\boxed{\text{False.}}$ The strictness of the NE matters.

e.g

	s	$\hat{s}$
s	1,1	0,0
$\hat{s}$	0,0	0,0

both s and $\hat{s}$ are NEs. But in this example, $u(\hat{s},\hat{s})$ is not strictly greater than $u(s,\hat{s})$. So in fact $\hat{s}$ is not an ESS even though it is a weak NE.

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Question 2 (30 marks)

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a)

roger \ caleb	P	NP
C	$8-x$, 0	$3-x$, 1
NC	4, 4	2, 3

P ≡ Caleb goes to party
NP ≡ Caleb doesn't go
C ≡ Roger hires clown
NC ≡ Roger doesn't hire.

b)

	P	NP
C	8, 0	3, $\boxed{1}$
NC	4, $\boxed{4}$	2, 3

Dominated: No clown (strict). Regardless of whether Caleb goes, Roger's payoff from C is strictly greater (8 vs 4, 3 vs 2).

NE (strict): (C, NP). i.e Roger's BR if Caleb NP is C, and Caleb's BR if Roger C is NP.

Payoffs (3, 1) for (Roger, Caleb).

c)

	P	NP
C	6, 0	1, $\boxed{1}$
NC	4, $\boxed{4}$	2, 3

Dominated: none.

No pure NEs.

Mixed NEs

	P	NP	
C	6, 0	1, $\boxed{1}$	p
NC	4, $\boxed{4}$	2, 3	$(1-p)$
	q	$(1-q)$.	

To find Caleb's mixed NEs, use Roger's indifference.

$$6q + (1-q) = 4q + 2(1-q) \Rightarrow q = 1/3$$

To find Roger's, use Caleb's indifference

$$4(1-p) = p + 3(1-p) \Rightarrow p = 1/2$$

$\therefore$ NE: $\left(\left(\frac{1}{3}, \frac{1}{2}\right), \left(\frac{1}{3}, \frac{2}{3}\right) \right)$, payoffs $\left(\frac{8}{3}, 2\right)$.

d)

	P	NP	
C	5, 0	0, $\boxed{1}$	p
NC	4, $\boxed{4}$	2, 3	$1-p$
	q	$1-q$	

Dominated: none.
No pure NEs.

Mixed NEs: $5q = 4q + 2(1-q) \Rightarrow q = 2/3$

$$4(1-p) = p + 3(1-p) \Rightarrow p = 1/2$$

NE: $\left(\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{2}{3}, \frac{1}{3}\right) \right)$, payoffs $\left(\frac{16}{3}, 2\right)$

e)

	P	NP
C	3, 0	-2, $\boxed{1}$
NC	4, $\boxed{4}$	2, 3

Dominated: Clown (strict): 4 vs 3, 2 vs -2.
NE (strict): (NC, P).
Payoffs (4, 4)

Question 3 (30 marks)

26/30.

Let $s_i \in \{1, 0\}$, $s_i = 1$ corresponds to vaccination

Let $m \equiv \sum_{i=1}^{n} s_i$ be the number of people vaccinated

$$u_i(s) = \begin{cases} 6 - i & , s_i = 1 \\ 6 \cdot \frac{m}{6} & , s_i = 0 \end{cases}$$

$$\Rightarrow u_i(s_i) = (6-i)s_i + m(1-s_i)$$

a) Possible deviations: i) 5 or 6 gets vaccinated

ii) 1-4 don't vaccinate

8/8

i) $u_5(1) = 1$, $u_5(0) = 4$ so player 5's BR is no vaccination.

$u_6(1) = 0$, $u_6(0) = 5$, so player 6 BR is also no vac.

ii) $u_4(1) = 2$, $u_4(0) = 3$. so actually player 4's BR is no vac.

Hence $s = (1,1,1,1,0,0)$ is not an NE. ✓

b) • We have already checked deviations where 4-6 get vaccinated.

• Now w check deviation where 1-3 don't vaccinate.

$$\left.\begin{array}{ll} u_3(1) = 3 & u_3(0) = 2. \\ u_2(1) = 4 & u_2(0) = 2 \\ u_1(1) = 5 & u_1(0) = 2 \end{array}\right\} \text{in each case, BR is to vaccinate.}$$

Since there are no profitable deviations, we conclude that each choice in $s = (1,1,1,0,0,0)$ is a BR to the others so this s is a NE.

8/8

c) • $s_1 = 1$ is weakly dominant. $u_1(1) = 5$. If all other players choose 1, her expected payoff is $\frac{5}{6} \times 6 = 5$.

• Now $s_i = 1$ cannot be weakly dominant strategies for other players, since $u_i(1) < 5$ for $i = 2, 3, \ldots, 6$, while the payoff if all other players choose 1 remains at 5

• What about $s_i = 0$? For player 6, $s_6 = 0$ is weakly dominant because $u_6(1) = 0$ while $u_6(0) \geq 0$ with equality when $s_i = 0 \; \forall i$. This is not true for $i \leq 5$ because they get ≥ 1 payoff from vaccinating while possibly zero payoff if $s_i = 0 \; \forall i$.

6/6

d) · Delete $s_1 = 0$ and $s_6 = 1$ since weakly dominated. ✓

· $u_2(1) = 4$, $u_2(0) \leq 4$. So $s_2 = 0$ weakly dominated. ✓

· $u_5(1) = 1$, $1 \leq u_5(0) \leq 4$. $s_5 = 1$ weakly dominated,

· Delete $s_2 = 0$, $s_5 = 1$. $\{1, 1, s_3, s_4, 0, 0\}$.

· $u_3(1) = 3$, $2 \leq u_3(0) \leq 3$.

· $u_4(1) = 2$, $2 \leq u_4(0) \leq 3$. i.e $s_4 = 1$ weakly dominated.

dubious.

3/4

e) · After iterative deletion, we have $s = \{1, 1, s_3, 0, 0, 0\}$.

· We can check pure NEs :

↳ $s = \{1, 1, 0, 0, 0, 0\}$ is not a NE since $s_3 = 1$ is better.

↳ only pure NE is $\{1, 1, 1, 0, 0, 0\}$ //.

1/4.

Check mixed NEs.

↳ incompatible for player 1 and player 6 to place any weight on their dominated strats. Likewise for 2 and 5.

Hence situation reduces to :

3 \ 4	V	D	
V	3, 2	3, 3	p
D	3, 2	2, 2	$(1-p)$
	q	$(1-q)$	

By indifference, the only mixed NE is $((0,1), (1,0))$

i.e $s = \{1, 1, 1, 0, 0, 0\}$.