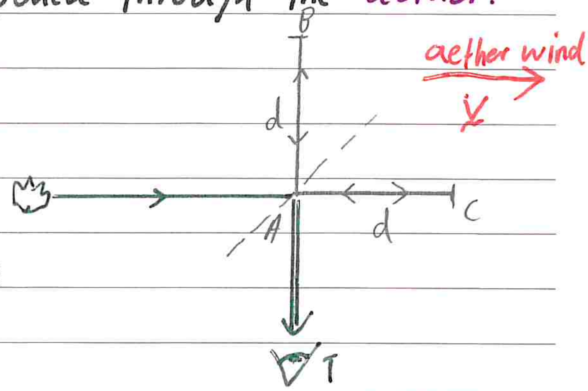
1A Special Relativity

No. 1
Date 2 . 4 . 19

- Classical physics proposed that light travelled through the aether.
- The Michelson-Morley experiment tested this using an interferometer.



↳ if there was aether:

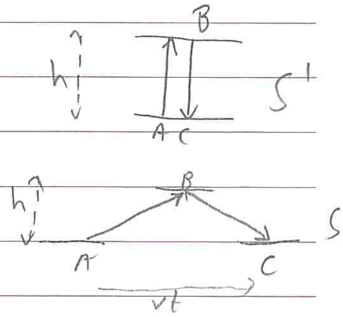
$$t_{ACA} = \frac{d}{c+v} + \frac{d}{c-v}, \quad t_{ABA} = \frac{2d}{\sqrt{c^2-v^2}}$$

$$\Rightarrow \Delta t \approx \frac{dv^2}{c^3}$$

↳ but there was no shift in observed $\Rightarrow$ no aether.

- Maxwell knew that $c = 1/\sqrt{\varepsilon_0 \mu_0}$, i.e depends on constants unrelated to the motion of the observer.
- Einstein's postulates:
  1. Speed of light (in vacuum) is the same for all observers
  2. Principle of relativity: laws of physics are the same in all inertial frames
- An important consequence is the loss of simultaneity: events that are simutaneous in one frame may not be in another frame.

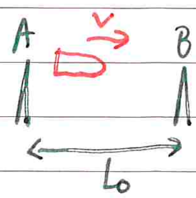
Time dilation



- Consider a light clock on a train:
  - in the clock's rest frame, $\Delta t'_{AC} = \frac{2h}{c}$
  - in the lab frame, $\Delta t_{AC} = \frac{2h}{\sqrt{c^2-v^2}}$

$$\Rightarrow \Delta t = \gamma \Delta t', \quad \gamma \equiv \frac{1}{\sqrt{1-v^2/c^2}}$$

- Because $\gamma > 1$, the time interval in the moving clock is lower when viewed from a stationary frame: 'moving clocks run slow'.
- The proper time interval is the shortest time interval between two events, measured by a clock present at both events



- Consider two beacons separated by L_0 in their rest frame, and a space ship travelling between them.
  - ship captain measures proper time $\Delta t'_{AB}$
  - observer at rest measures Δt_{AB}

- The observer sees $\Delta t_{AB} = L_0/v$
- The captain sees $\Delta t'_{AB} = L/v$

$$\Rightarrow L = \frac{L_0}{\gamma}$$ i.e objects are longest in their rest frame (proper length). Length contraction otherwise

- There is no length contraction perpendicular to motion.

Evidence for SR

- Time-dilation in the decay of muons: Earth-based clocks measure sufficiently long time intervals for the muons to reach the surface before decaying
- Michelson-Morley: c invariant
- GPS satellites require a correction

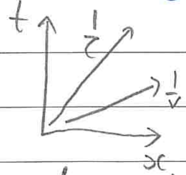
Relativistic kinematics

- Lorentz transformation:

$$\Delta x' = \gamma(\Delta x - v\Delta t) \qquad \xleftrightarrow{\text{flip primes}} \qquad \Delta x = \gamma(\Delta x' + v\Delta t')$$

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) \qquad \text{flip signs} \qquad \Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right)$$

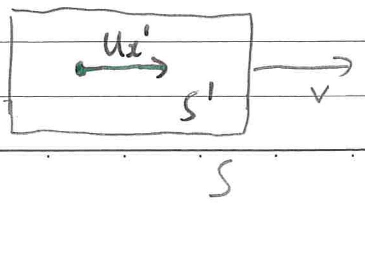
- It is often helpful to draw spacetime diagrams (one for each frame).



- While the Lorentz transforms have c as an upper bound, this does not apply to virtual objects (e.g shadows).

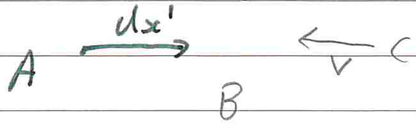
Velocity addition

- Object moving at u_x' relative to S', but S' moving at v relative to S



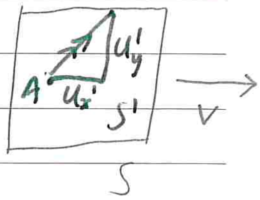
$$u_x = \frac{\Delta x}{\Delta t} = \frac{\Delta x' + v\Delta t'}{\Delta t' + v\Delta x'/c^2} \Rightarrow u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}$$

- This formula only applies when finding the velocity of the object relative to the stationary frame as measured by the object or S



← relative speed of A and C measured by B is just $v + u_x'$

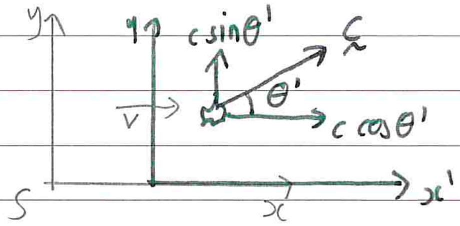
- If the motion has a transverse component, that will be affected because of time dilation



$$u_y \equiv \frac{\Delta y}{\Delta t} = \frac{\Delta y'}{\gamma(\Delta t' + v\Delta x'/c^2)} \Rightarrow u_y' = \frac{u_y'}{\gamma(1 + u_x' v/c^2)}$$

Aberration of light

- It is observed that light from distant stars appears to come in at a slight angle → originally used as evidence for aether.
- Consider a star stationary w.r.t S', but S' moving at speed v w.r.t S. We can apply velocity addition to the photon's components:



$$\underset{\sim}{c} = (c_x, c_y) = \left(\frac{c\cos\theta' + v}{1 + v\cos\theta'/c}, \frac{c\sin\theta'}{\gamma(1 + v\cos\theta'/c)} \right)$$

- But in frame S, $c_x = c\cos\theta$ and $c_y = c\sin\theta$

$$\therefore \cos\theta' = \left(\frac{\cos\theta - \frac{v}{c}}{1 - \frac{v}{c}\cos\theta} \right)$$ and similar for $\sin\theta'$

- Thus the stellar aberration can be explained by SR.

Doppler effect

- Consider a signal with freq f' emitted by an object moving towards you at speed v. $\Delta t' = 1/f'$
- In your frame, the photons have travelled $c\gamma\Delta t'$ and the source has travelled $v\gamma\Delta t'$ by the next flash.
- $\therefore$ photons reach your eye with $\Delta T = \frac{1}{c}((c-v)\gamma\Delta t'$

$$\Rightarrow f = f'\sqrt{\frac{1 + v/c}{1 - v/c}}$$

- Because of time dilation, there will also be a transverse Doppler shift even when the signal is perpendicular to motion.

Relativistic mechanics

- The relativistic momentum of a particle with velocity $\underline{v}$ is given by $\underline{p} = \gamma_v m v$
- The total energy of a relativistic particle is $E = \gamma m c^2$
- The KE of a particle is the total E − rest mass E: $K = (\gamma - 1) m c^2$
- $\underline{p}$ and E conserved, but K is not.
- For a single particle:
  ↳ $E^2 - p^2c^2 = m^2c^4$
  ↳ $K^2 + 2Kmc^2 = p^2c^2$
- The E-p invariant states that the quantity $E^2 - p^2c^2$ is constant in all inertial frames. E and p are the totals for the system, including sign in the case of $\underline{p}$.
- Energy and momentum transform in the same way as spacetime, substituting $(ct, x, y, z) \rightarrow (E/c, p_x, p_y, p_z)$.

$$\therefore \quad E'/c = \gamma(E/c - v p_x/c)$$
$$p_x' = \gamma(p_x - vE/c^2)$$