E. Mechanical Behaviour

No. 1
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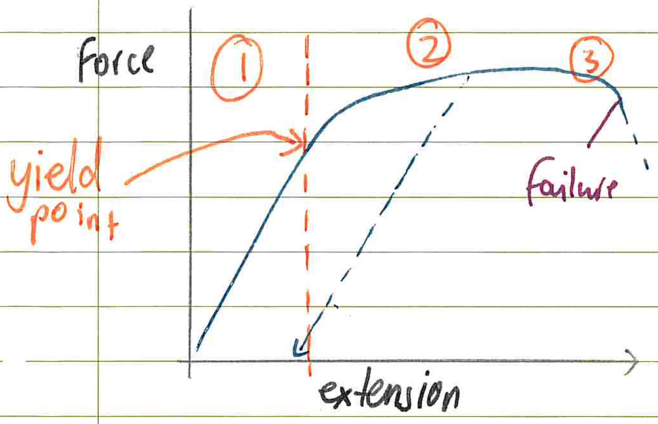
- A material of known dimensions can be examined via tensile testing, in which it is stretched.

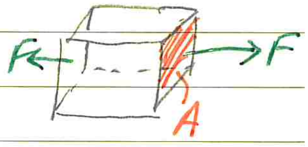


① Elastic region – deformation is linear and fully recoverable

② Plastic deformation – permanent shape change even when external load has been removed

③ Necking – sample gets thinner so less force needed to extend.

- Force and extension depend on a material's dimensions. It is more useful to have quantities that depend only on the material.
- The normal stress σ is the outward force per unit area on a cuboid: $\sigma = F/A$



↳ by definition, $\sigma > 0 \iff$ stretching
$\iff$ tensile stress

↳ $\sigma < 0 \Rightarrow$ compressive stress

- Stress on a material leads to normal strain ε, the relative change in the linear ~~direction~~ dimension in the direction of the force.

$$\delta\varepsilon = \frac{\delta l}{l} \Rightarrow \varepsilon_{true} = \int_{l_0}^{l_i} \frac{dl}{l} = \ln(l_i/l_0).$$

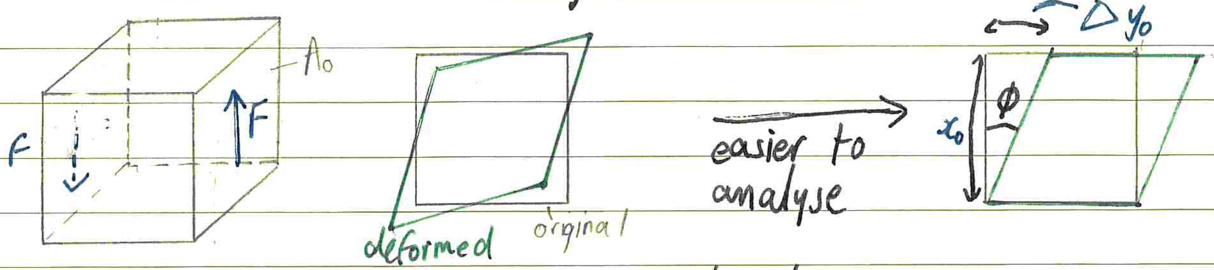
↳ this can be approximated by the engineering strain ε_{eng}, which is the % change in length, when ε_{true} is small.

- The engineering stress assumes that the area of the cross-section remains the same even when strained, i.e $\sigma_{eng} = F/A_0$.
  ↳ reasonable approximation when strain < 2% because most engineering focuses on the elastic region.
- In reality, a cuboid under tension will contract laterally
  - Poisson's ratio ν is the ratio of ~~axial~~ lateral contraction to axial elongation: $\varepsilon_x = \varepsilon_y = \nu\varepsilon_z$
  - most materials have ν in the range 0.2–0.5
  - cork has $\nu \approx 0$; when forced into bottle it doesn't expand.

- A force applied parallel to a surface results in a shear stress

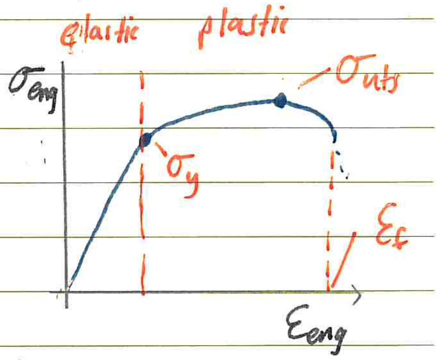
$$\tau = \frac{F}{A_0}$$ ← sign of τ has no physical meaning.

- Shear stress causes one diagonal axis to shorten and one to lengthen.



- The shear strain is the distortional deviation:

$$\gamma = \frac{\Delta y_0}{x_0} = \tan\phi$$ ↲ angle of shear



- The stress-strain curve can be plotted:
  - yield stress σ_y is the maximum stress before plastic deformation (a.k.a strength).
  - ultimate tensile strength σ_{uts} represents the maximum load a material can bear
  - strain to failure ε_f is the strain a material can sustain before failing.

Elastic deformation

- In the elastic region, the strain is directly proportional to stress.
- For normal stress, the constant of proportionality is Young's Modulus E

$$\sigma = E\varepsilon$$

- For shear stress, the equivalent is the shear modulus G: $\tau = G\gamma$
- Both E and G are measured in pascals
- Most structural materials have $E \sim 100-200\,GPa$
- Composite materials are more complex, and can have moduli that depend on the axis being tested.
- In the elastic region, behaviour is exactly analogous to a spring.

- Because elastic deformation is recoverable, the energy must be stored somewhere.
- Stretching a sample by dL requires work to be done:

$$dW = F\,dL = (\sigma A)(L\,d\varepsilon)$$

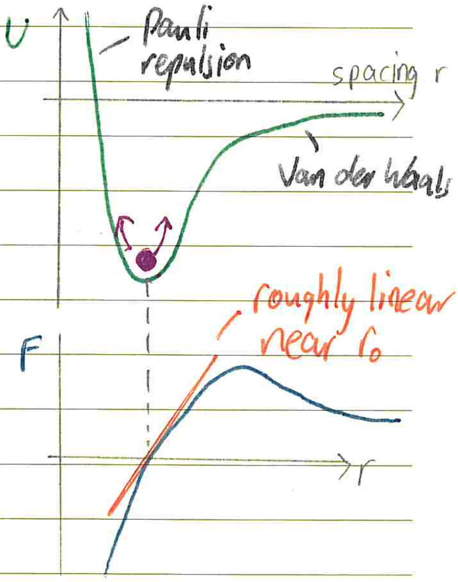
i.e $dW = VE\varepsilon$, assuming $V \equiv AL$ is constant.

$$\therefore W = \int_0^{\varepsilon_{max}} VE\varepsilon\,d\varepsilon$$

$\Rightarrow$ work done per unit volume $= \frac{1}{2}E\varepsilon^2$

- Likewise, the work/volume for a shear stress is $\frac{1}{2}G\gamma^2$

- At an atomic level, elasticity is the result of a near-quadratic potential.
  - for very small separations, (neutral) atoms experience a Pauli repulsion
  - however, in general there is the Van der Waals attraction.
  - there will be some equilibrium separation r_0 that minimises the energy.



- Can be modelled by the Lennard-Jones potential:

$$U_{LJ} = U_0\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^6\right]$$

↳ If we model each bond as having area r_0^2:

$$\sigma = \frac{E}{r_0^2} = \frac{1}{r_0^2}\frac{dU}{dr}.$$

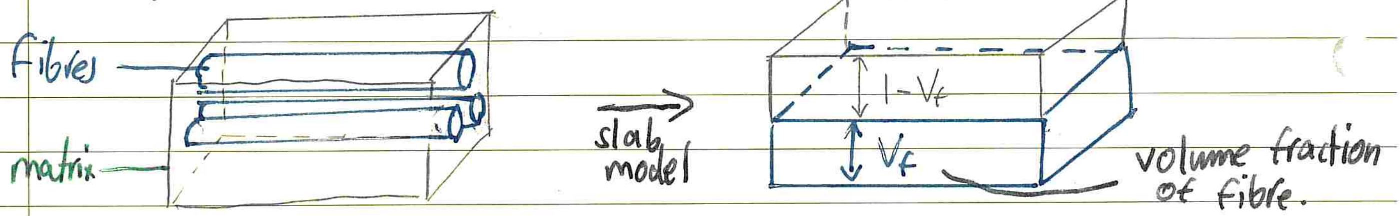
↳ combined with $d\varepsilon = dr/r_0$:

$$E = \frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{dr}\frac{dr}{d\varepsilon} \Longrightarrow E = \frac{1}{r_0}\left.\frac{d^2U}{dr^2}\right|_{r=r_0}$$

↳ i.e Young's modulus is equal to the curvature of the potential well about equilibrium.

Composite materials

- Combining materials can result in 'best of both worlds' properties
- Typically, a ceramic fibre is embedded in a polymer matrix, e.g glass/carbon/aramid fibres in epoxy.
- These are strongest when fibres are parallel and aligned, but this leads to highly anisotropic elastic behaviour, so in practice plies of aligned fibre are stacked in different directions.
- Composites can be modeled as slabs of fibre / matrix

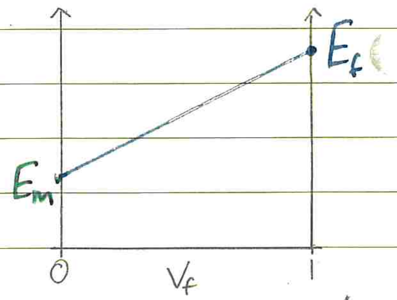


- The axial modulus can be approximated by the Voigt Model, which assumes that each slab experiences the same normal strain.

stress on composite $\rightarrow$ $\sigma_c = \sigma_f V_f + \sigma_m V_m = \sigma_f V_f + \sigma_m (1-V_f)$

$$\therefore \boxed{E_c = E_f V_f + E_m (1-V_f)}$$ using the Voigt assumption i.e $\varepsilon_c = \varepsilon_f = \varepsilon_m$.

- This results in a linear rule of mixture (i.e weighted average) that provides reasonable estimates of E_c.



- In the case of the transverse modulus, we assume that the slabs are under the same ~~stra~~ stress but can extend differently.

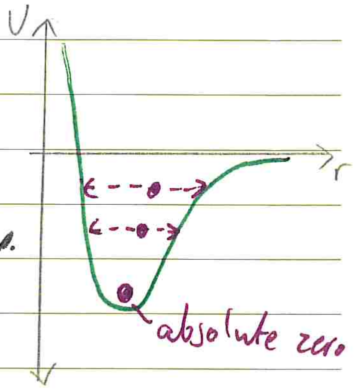
$$\varepsilon_c = \varepsilon_f V_f + \varepsilon_m V_m \implies \frac{1}{E_c} = \frac{V_f}{E_f} + \frac{(1-V_f)}{E_m}$$

- This model produces inaccurate values → lower bound estimate only.
  - ↳ parts of the matrix near the fibres are shielded from stress because the fibres are very stiff vertically
  - ↳ model overestimates matrix extension, underestimates E_c.

Thermal strains

- The temperature measures the mean KE of atoms:

$$\frac{3}{2}kT = \langle \tfrac{1}{2}mv^2 \rangle$$



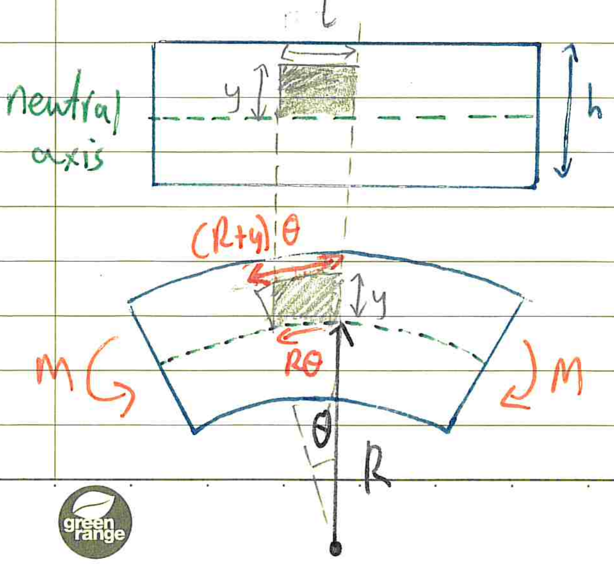
- Thus at higher temperatures, atoms are no longer stationary at the equilibrium position.
- The asymmetry of the potential well means that the mean ~~position~~ separation increases with temp.
  ↳ materials expand when heated.
- This results in a near-linear relationship between thermal strain and temperature:

$$\varepsilon_T = \alpha \Delta T,$$ where α is the coefficient of thermal expansion.

- Materials with stronger inter-atomic bonds have deep sharp potential wells ⇒ high E, low α. e.g ceramics.
- Polymers have shallow potential wells ⇒ low E, high α.
- α itself has a small increasing trend with temperature, but this rate of change depends on the specific potential well.
- As temp ↑, the average d^2V/dr^2 decreases ⇒ $E\downarrow$.
- Thermal strain can be applied in bimetallic strips

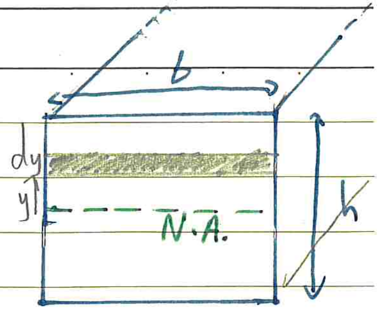
Beam theory

- Consider a uniform beam subject to pure bending (i.e no shear)



- The top will be subject to tension and the bottom to compression, but there will be some undeformed neutral axis
- This helps us define the radius of curvature, $R = L/\theta$.

$$\varepsilon_{axial} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R} \Rightarrow \boxed{\sigma_{axial} = \frac{Ey}{R}}$$



- The force on a cross-sectional element (looking down the beam) is $F=\sigma A=\frac{Ey}{R}\cdot b\,dy$
  - ↳ in general, $b=b(y)$ for non-cuboids.
- Moment about the neutral axis:

$$M_y = y\left(\frac{Ey}{R}\right)\cdot b(y)dy \leftarrow \text{force} \times \text{distance.}$$

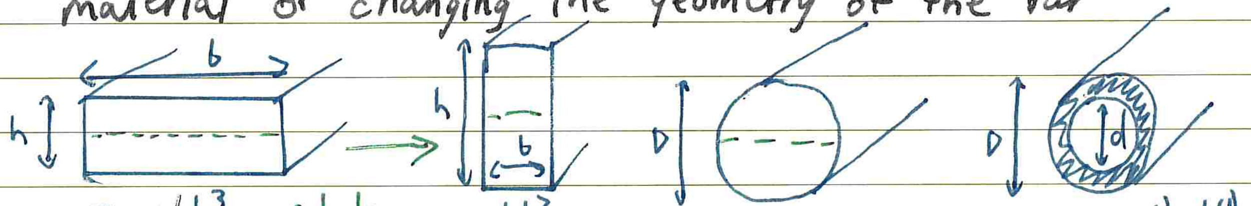
- Total bending moment found by integrating:

$$M=\frac{E}{R}\boxed{\int_{-h/2}^{h/2} y^2 b(y)dy} \leftarrow \text{second moment of area } (I)$$

- Can be written in terms of E, I, and curvature $\kappa = 1/R$

$$\boxed{M=\kappa EI} \rightarrow EI \text{ is the flexural rigidity (ie beam stiffness).}$$

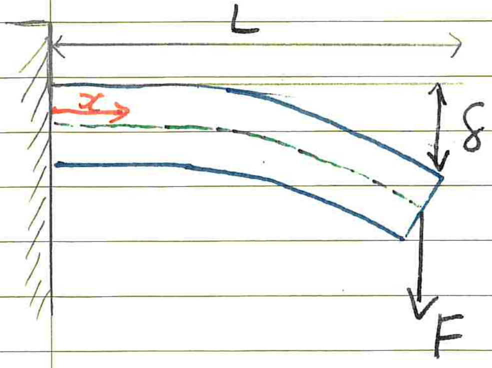
- Flexural rigidity can thus be increased by using a stiffer material or changing the geometry of the bar



$I=\frac{bh^3}{12}$ rotate to ↑I. $I=\frac{bh^3}{12}$ $I=\frac{\pi D^4}{64}$ → $I=\pi\frac{(D^4-d^4)}{64}$

- Using the principle of superposition, $I_{hollow}=I_{solid}-I_{void}$

Cantilever beams



- Bending moment is function of x:

$$M=F(L-x)$$

- Consider a small segment: $R\,d\theta \approx dL \approx dx \Rightarrow \frac{d\theta}{dx}=\frac{1}{R}$
- But $\theta \sim dy/dx$

$$\therefore \boxed{\kappa=\frac{1}{R}=\frac{d^2y}{dx^2}}$$

- Thus the equation for cantilever loading: $\boxed{M=EI\frac{d^2y}{dx^2}=F(L-x)}$.

- This equation can be solved for y using $y'(0) = y(0) = 0$ as boundary conditions (horizontal at $x=0$).

$$\therefore \; y = \frac{Fx^2}{6EI}(3L - x)$$

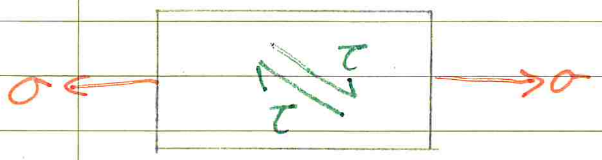
- The max deflection is obviously at the end of the beam $(x = L)$

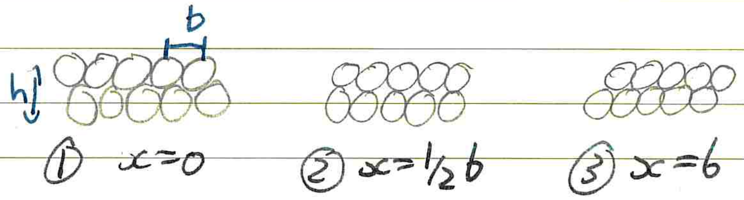
$$\Rightarrow \; \boxed{\delta_{max} = \frac{FL^3}{3EI}}$$

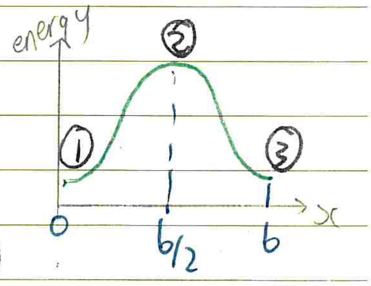
higher flex rigidity ⇒ lower δ.

Plastic deformation

- Plastic deformation involves a change at the atomic level which is irreversible.
- Viewing a plasticly deformed substance under a microscope shows fine parallel lines at an angle to the applied stress
- The simplest model considers planes of atoms which only move when a critical value of the shear stress has been applied.







- Clearly, energy is lowest in ① and ③ (stablest) and maximal at ②
- Thus the shear stress must be sinusoidal

i.e $\tau = C\sin\left(\frac{2\pi x}{b}\right)$

↳ using small $\frac{x}{b}$ and $\gamma = x/h$ (shear strain)

$$\tau \approx C\left(\frac{2\pi h\gamma}{b}\right), \text{ but } \frac{\tau}{\gamma} = G \Rightarrow C = \frac{Gb}{2\pi h}.$$

↳ C is the maximum value, i.e $\tau_{crit} = C$

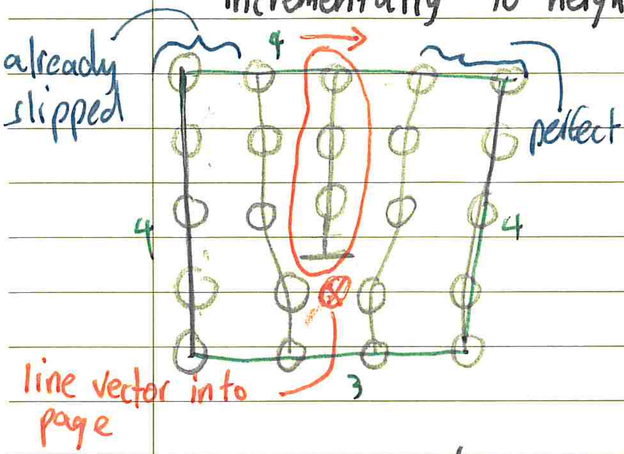
$$\Rightarrow \; \tau_{crit} = \frac{Gb}{2\pi h}$$

- This model overestimates τ_{crit} by a factor of ten

↳ i.e there must exist an easier way for planes to slip.

Dislocations

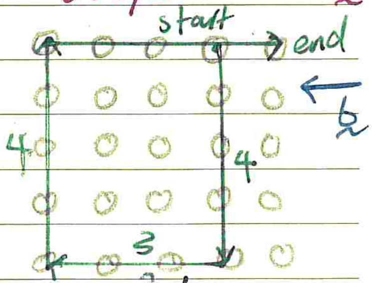
- An alternative theory models slip as the movement of local structural defects called dislocations.
- When sufficient stress is applied, the dislocation can move incrementally to neighbouring atoms



- Consider an extra half-plane of atoms moving to the right – this is an edge dislocation, i.e. there is a line going into the page which joins dangling bonds.
- This line separates the slipped and perfect parts of the crystal and defines the line vector $\underset{\sim}{l}$ for an edge dislocation.

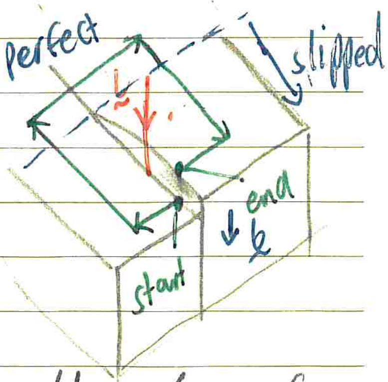
- The magnitude and direction of the lattice displacement caused by a dislocation is described by the Burgers vector $\underset{\sim}{b}$
  - found by constructing a Burgers circuit around the dislocation core then repeating the integer steps in a perfect crystal
  - there will be a closure failure: $\underset{\sim}{b}$ goes from the finish to start of the path

use RH rule around line vector



- Dislocation lines cannot end arbitrarily – they either terminate at a boundary or form a closed loop.

- Screw dislocations involve a partial shear of the crystal: the Burgers circuit is now a 3D path → can be thought of as tracing a square helix with the line vector through the middle.



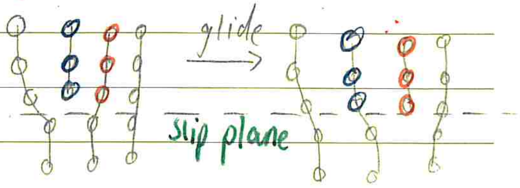
- The relationship between $\underset{\sim}{l}$ and $\underset{\sim}{b}$ determines the character of the dislocation:

$\underset{\sim}{b} \perp \underset{\sim}{l} \iff$ edge dislocation

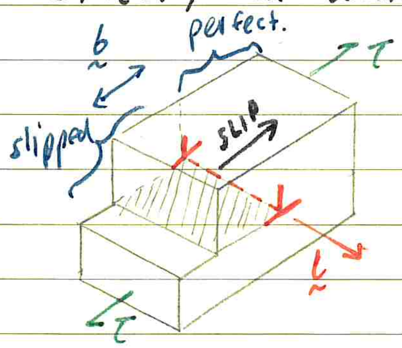
$\underset{\sim}{b} \parallel \underset{\sim}{l} \iff$ screw dislocation

Dislocation motion

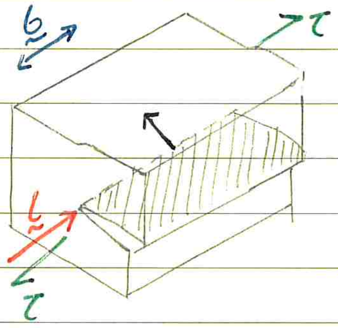
- Only small shifts of some atoms are required for dislocation movement.
  - when a critical shear stress τ_{crit} is applied, the half plane is displaced such that it is favourable to glide one increment
  - glide occurs on a slip plane, which contains both Burgers and line vectors.
  - the slip direction is parallel or antiparallel to $\underset{\sim}{b}$, and atoms are displaced by $|\underset{\sim}{b}|$
  - the original crystal structure is restored after passage.



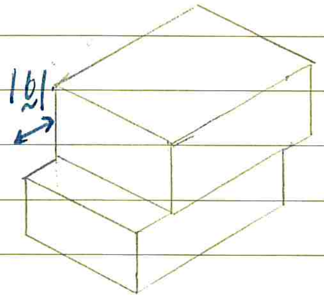
- Both edge and screw dislocations lead to the same macroscopic effect, but with a different mechanism.



EDGE



SCREW



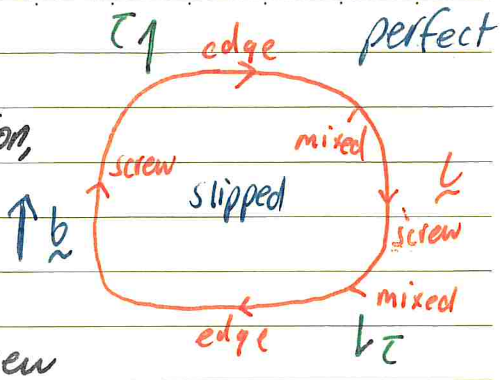
Result

- If the shear stress is aligned with $\underset{\sim}{b}$ and $\tau \geq \tau_{crit}$, slip occurs parallel to the Burgers vector and the dislocation moves perpendicular to its line vector.
  - for an edge dislocation, there is only one slip plane (because it must contain $\underset{\sim}{b}$ and $\underset{\sim}{l}$, with $\underset{\sim}{b} \perp \underset{\sim}{l}$).
  - but there are many possible slip planes for screw dislocations.

Dislocation loops

- A dislocation loop consists of a central slipped area surrounded by an unslipped region. e.g formed when ~~the~~ part of the top layer of a crystal is diagonally displaced.

- The dislocation line is thus a loop, with $\underset{\sim}{l}$ changing direction
- But because it is still a single dislocation, there is only one Burgers vector
  - ↳ character of dislocation changes around the loop.
- Edge dislocations move // τ, while screw dislocations move ⊥ τ.
- Because the line vectors on opposite sides of the loop are antiparallel, the segments are of opposite sense, so the loop expands.



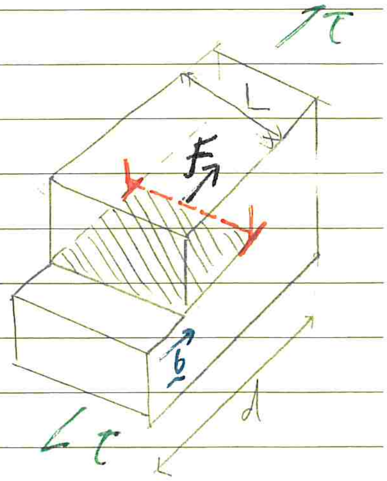
Stress and work to move dislocations

- Although ⊥ movement requires much less stress than block slip, we must apply enough to move it through a high energy state.
- This is described by the Peierls-Nabarro stress which is associated with the Peierls-Nabarro energy. τ_p is hard to calculate exactly, but can be modeled as:

$$\boxed{\tau_p \approx 3G\exp\left(-\frac{2\pi w}{b}\right)}$$

  - ↳ w is the dislocation width, i.e. the distance over which atoms are displaced by more than $\approx |\underset{\sim}{b}|/4$.
  - ↳ w has a lower bound of $|\underset{\sim}{b}|$, so τ_p is maximised when $w = b$. Much less stress is required for wide dislocations (high w/b).

- The work done to shear a crystal a distance $|\underset{\sim}{b}|d$ is given by $W = \tau L d |\underset{\sim}{b}|$
- We can use this to define a glide force, the force per unit length to move a dislocation.

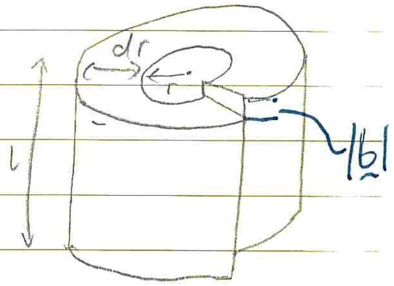


$$\tau L d |\underset{\sim}{b}| = F L d$$

$$\Rightarrow \boxed{F = \tau |\underset{\sim}{b}|}$$

Energy of a dislocation

- Dislocations create elastic strain in a crystal, which has an associated strain energy.
- This can be analysed for screw dislocations by considering an annular cylindrical element



$$dV = 2\pi r l\, dr$$

$$\gamma = b/2\pi r \quad \text{(unwrap into cuboid)}$$

↳ $dU = \frac{1}{2} G \gamma^2 dV = \frac{1}{2} G \left(\frac{b}{2\pi r}\right)^2 (2\pi r l\, dr)$

↳ at the core of the dislocation, energy will be too great to be treated as an elastic strain energy. Hence we call it E_{core} and only integrate from some r_0.

↳ $U = \int dU = E_{core} + \frac{Gb^2 l}{4\pi} \int_{r_0}^{r_\infty} \frac{1}{r} dr$

$$\Rightarrow \boxed{U = \frac{Gb^2 l}{4\pi} \ln\left(\frac{r_\infty}{r_0}\right) + E_{core}}$$

↳ r_0 is chosen to overestimate the energy near r_0 in order to compensate for E_{core}. Empirically, $\boxed{r_0 \approx |b|/4}$

↳ r_∞ is set to half the distance between dislocations.

↳ together, this gives the strain energy per unit length Λ

$\boxed{\Lambda \approx \frac{1}{2} G|b|^2}$ i.e favours shorter $|b|$ and thus prefers close-packed slip.

- The energy for edge dislocations is always greater than for screw (but harder to derive).

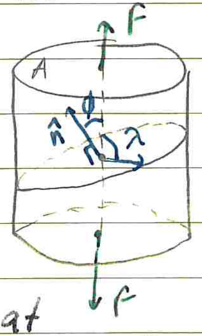
Slip in real crystals

- For slip to occur, the Burgers vector must correspond to a lattice vector. Smaller $|\underset{\sim}{b}|$ preferred, favouring slip in close-packed directions.
- ~~Lower~~ ↑width ⇒ less stress to slip. Thus slip is favoured on low index close-packed planes, which have large interplanar spacings and are densely packed.
- A slip system is the combination of a slip plane and direction.
- The number of slip systems is determined by crystal structure.

	bcc	fcc	hcp
Slip system	$\{110\}\langle\bar{1}11\rangle$	$\{111\}\langle\bar{1}10\rangle$	$\{001\}\langle100\rangle$
number	$6 \times 2 = 12$	$4 \times 3 = 12$	$1 \times 3 = 3$
$\underset{\sim}{b}$	$\frac{a}{2}\langle111\rangle$	$\frac{a}{2}\langle110\rangle$	$a\langle100\rangle$

- This explains why hcp metals are stronger but less ductile – there are fewer slip systems. ~~(intuitive given ABCABC packing).~~
- fcc-like → NaCl structures: $\{110\}\langle\bar{1}10\rangle$ $(6 \times 1 = 6)$
- bcc-like → CsCl structures: $\{110\}\langle001\rangle$ $(6 \times 1 = 6)$

- For slip to occur, there must be a component of stress parallel to the Burgers vector. Thus not all dislocations are mobile.
- Consider a stress applied to a single crystal:
  - ↳ area of plane is $A/\cos\phi$
  - ↳ force // to slip direction is $F\cos\lambda$



$$\tau_R = \frac{F\cos\lambda}{A/\cos\phi} = \frac{F}{A}\cos\phi\cos\lambda$$ (Schmid's law)

- The critical resolved shear stress is the value of τ_R at which slip occurs:

$$\boxed{\tau_c = \sigma_y \overbrace{\cos\phi\cos\lambda}}$$

→ Schmid factor $\in [0,1]$

→ because σ_y is the F/A for which dislocations move

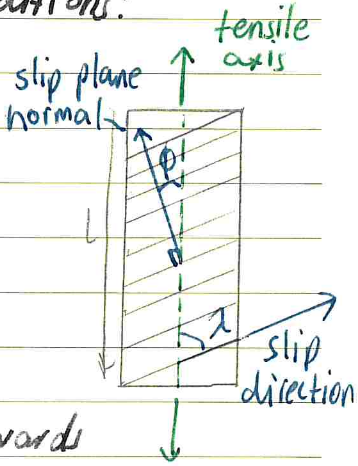
- Slip will first occur for the system with the highest Schmid factor.
- For bcc and fcc, we can use the OILS rule as a shortcut
  1. Write down tensile axis [U V W]
  2. Find Highest, Intermediate, Lowest indices (ignoring sign)
  3. If fcc:
     - slip is ⟨110⟩ with 0 in position of I index

     else if bcc:
     - slip plane is {110} with 0 at I.
  4. If fcc: slip plane is {111} with L sign reversed

     else if bcc: slip ⟨111⟩ with L sign reversed.

Geometry as slip proceeds

- Slip will encourage lateral displacement in a crystal under tension.
- But if the grips are aligned and fixed, there will be rotation of planes and local bending near the grips.
- This plastic deformation is controlled by two conditions:
  - interplanar spacing remains constant
  - number of planes (N) is conserved.



$$\therefore \quad l = \frac{Nd}{\cos\phi} \quad \text{or} \quad l = \frac{Nd}{\sin\lambda}$$

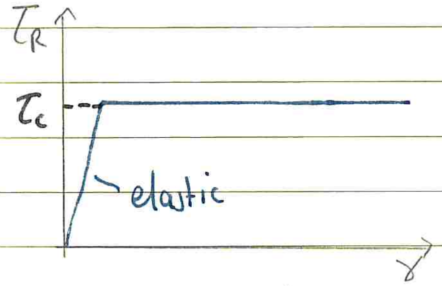
- So any elongation causes $\phi\uparrow$ and $\lambda\downarrow$
  ↳ Schmid factors change and new slip systems can activate
- In the sample frame, the tensile axis rotates towards the slip direction. After each slip, it rotates closer
  - e.g after n slips in fcc with original tensile axis $[\bar{2}14]$, the tensile axis will be $[\bar{2}14] + n[011]$
  - when two indices have the same value (ignoring sign), there will be a new slip system with the same Schmid factor

    e.g $n = 1 \Rightarrow TA = [\bar{2}25]$ so $(111)[\bar{1}01]$ activates

    $\therefore TA = [\bar{2}25] + n[011] + n[\bar{1}01]$
  - these slips take turns, but eventually $TA \rightarrow [\bar{1}12]$.
  - deformation along $[\bar{1}12]$ is purely elongation. Rotation cancels out.

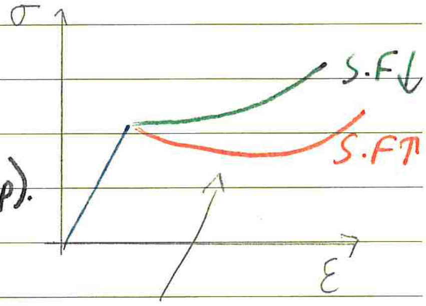
Plastic deformation of metallic crystals

- For an hcp metal, only one slip system can operate, so the critical resolved shear stress will not vary when deforming.



i.e once $\tau_R = \tau_c$, it remains there.

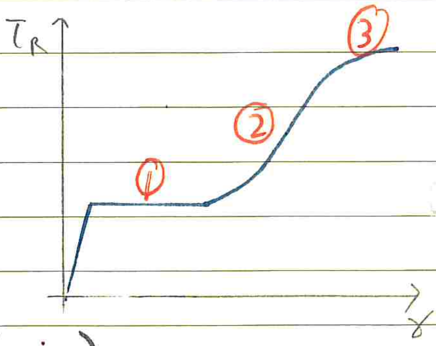
- But as the TA rotates, ~~its~~ the Schmid factor of the slip system changes (depending on the crystal's initial orientation).



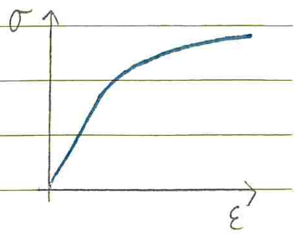
$$\sigma_y = \frac{\tau_c}{\cos\phi \cos\lambda}$$ (because τ_c constant for hcp).

↳ i.e if S.F↓ σ_y↑

↳ if S.F↑, σ_y↓ – this is geometric softening.



- In fcc metal, initially one slip system operates so τ_R is constant ①, though the Schmid factors change: easy glide
- In ②, duplex slip occurs. Dislocations move on both slip systems and can interact
  ↳ more τ_R needed to move dislocations
  ⇒ strength increases (work hardening).
- τ_R is large enough to activate new slip systems – dislocations can now bypass obstacles via cross slip.
  ↳ cross slip easier if stacking fault energy high ⇒ lower τ_R to reach ③



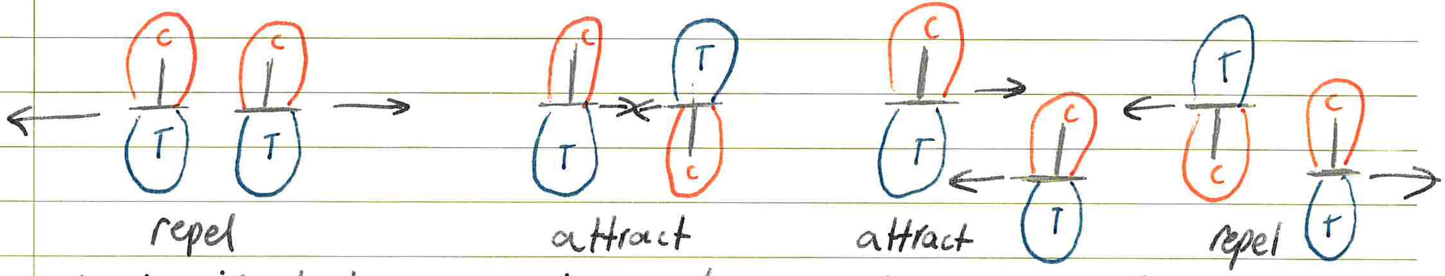
- In polycrystalline metal, crystals are randomly oriented with the mean Schmid factor as 1/3. ← Taylor factor
- Hence there is no easy glide, because duplex slip immediately occurs for some of the crystals.

→

- Using the Taylor factor, we expect $\sigma_y \approx 3\tau_c$ for polycrystalline materials. But this is an underestimate because it ignores grain boundaries.
  - ↳ deformation of a grain must be compatible with neighbours
  - ↳ so multiple slip is required ⇒ greater σ_y.

Dislocation interactions

- Dislocations strain the lattice. For screw dislocations, $\tau = G\frac{b}{2\pi r}$.
- Edge ⊥s are more complex, but can be modelled as causing compression above the slip plane (extra atoms), and tension below.
- Stress fields will interact so as to reduce strain



- ↳ i.e if ⊥ has same sign and same plane, it will repel
- ↳ try to overlay Ⓣ and Ⓒ
  - ↳ this can lead to ordered arrays of ⊥ forming.

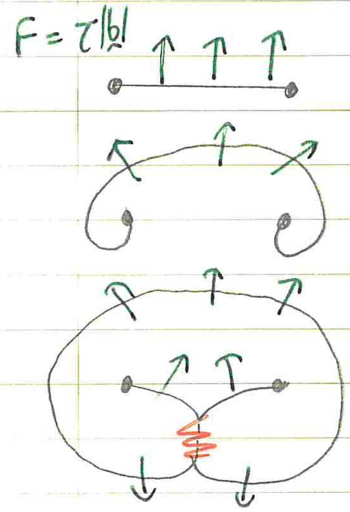
- If two dislocations were to combine, the new Burgers vector would be the vector sum of the individual $\underset{\sim}{b}$. $\underset{\sim}{b}_3 = \underset{\sim}{b}_1 + \underset{\sim}{b}_2$
- The new line vector would be the intersection of both slip planes.
- However, it is only energetically favourable for dislocations to combine if Frank's rule holds:

$$\boxed{b_3^2 < b_1^2 + b_2^2} \quad \leftarrow \text{because energy} \sim |\underset{\sim}{b}|^2$$

- Using the new $\underset{\sim}{b}_3$ and $\underset{\sim}{l}_3$, assuming Frank's rule holds and the dislocations combine, we can construct a slip ~~system for~~ plane for the new dislocation
  - ↳ unique for edge dislocation: only one plane contains $\underset{\sim}{b}_3$ and $\underset{\sim}{l}_3$
  - ↳ no guarantee that result will be on a valid slip system
    - ⇒ sessile dislocation that blocks other dislocations' movement.

Dislocation generation

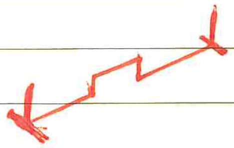
- Once dislocations reach the surface, they no longer participate in deformation ⇒ there must be a mechanism for ⊥ generation.
- A Frank-Read source consists of a linear dislocation pinned at both ends (e.g by a sessile dislocation or point defect).



- initially, ⊥ is a straight line with $\underline{b}$ normal
- when τ applied, ⊥ experiences $F = \tau|\underline{b}|$
- because it is pinned, it bows out to balance the line tension with the shear force.
- eventually it spirals round the pinned points.
- when the two arms meet below, they annihalate because the segments have opposite sense
- this creates a complete ⊥ loop and a new ⊥.
- if stress is maintained, the ⊥ loop will propagate outwards and the process repeats for the new ⊥.

Out of plane movement

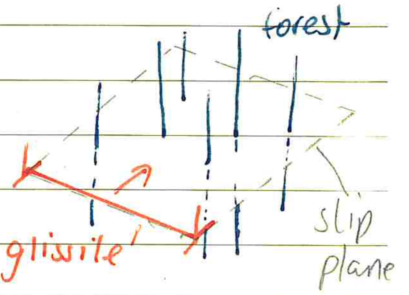
- Edge dislocations, for which glide is confined to a single plane, can bypass obstacles by climb - moving to a parallel slip plane
  - ↳ if a vacancy moves to the ⊥ core, ⊥ will move up - positive climb
  - ↳ alternatively, in negative climb, the vacancy at the ⊥ core moves downwards (i.e replaced by atom), so ⊥ moves down.
  - ↳ this produces jogs, steps in the ⊥ line that are not in the ~~sp~~ slip plane.



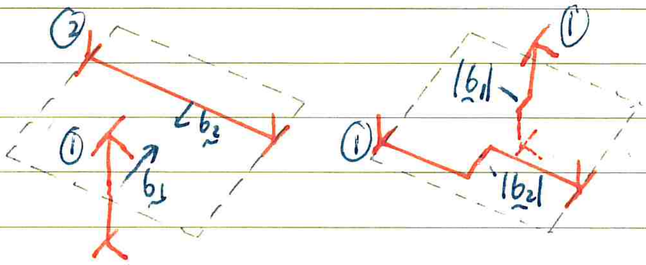
  - ↳ climb requires significant diffusion and is thus thermally activated.
- Screw dislocations can easily cross slip with sufficient τ_R
  - when obstacle encountered, τ_R builds until new slip system activates.
  - at this point, the screw dislocation can continue on the rotated plane. It may double cross slip back to the original plane if the Schmid factor is greater.

Strengthening single-phase metals

- Strengthening requires processing that reduces dislocation mobility, such that more stress is required for plastic deformation.
- The dislocation density ρ is the number of dislocations intersecting a unit area (total dislocation line length / unit volume).
- Assuming a cubic array of $\perp$ separated by L, $\rho = 1/L^2$
  - ↳ $\rho \sim 10^{10}\,m^{-2}$ annealed, $\sim 10^{16}\,m^{-2}$ for plastically deformed.



- Primary slip planes will be threaded with sessile dislocations (forest dislocations)
- Thus a glissile dislocation will have to intersect all the forest dislocations to traverse



- When the dislocations cut they form jogs whose lengths are equal to the intersecting $\underset{\sim}{b}$
- Because jogs increase $\perp$ length, it requires energy to cut.
- If the jogs are sessile, there will be strengthening as a result of further barriers to slip ⇒ forest hardening (a mechanism for work hardening)

Grain boundaries

- Dislocations cannot simply pass from one grain to the next: they accumulate at the boundary in a dislocation pile - yield only occurs when applied stress is significant
- But the pile itself causes stress because $\perp$s repel
  - ↳ large grains ⇒ long piles ⇒ more stress ⇒ lower yield stress
  - ↳ smaller grains ⇒ higher yield stress
- Related by Hall-Petch: $\boxed{\sigma_y = \sigma_0 + k/\sqrt{d}}$

constant
diameter

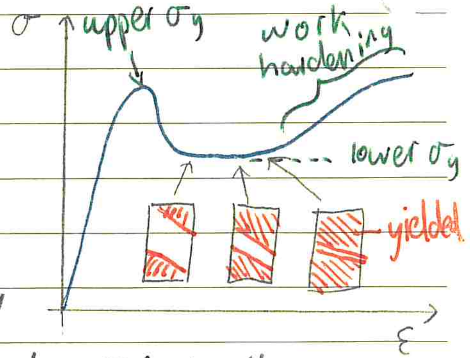
intrinsic yield stress

Solid solution strengthening

- Alloying tends to increase strength because it introduces solute atoms that interact with dislocations.
- Substitutional defects are when the solute atom replaces the original
  - produces a spherically symmetric stress field: compressive if solute atom is larger, tensile otherwise.
  - no shear stress component so don't interact with screw dislocations.
  - they will attract and repel edge dislocations, e.g large (compressive) defects will attract the tensile part of $\perp$
  - once an arrangement of defect / $\perp$ is formed, it will tend to persist, inhibiting further movement of dislocations (strength $\uparrow$).
  - but solute atoms are essentially static, so this will only happen if they are close to a slip plane.
  - thus substitutional solutes provide a weak strengthening effect.

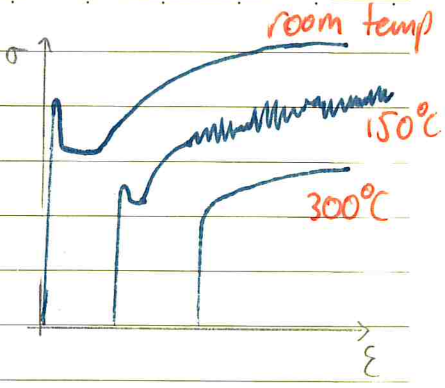
- Interstitial defects cause large compressive distortions, with asymmetric stress fields that interact with edge and screw dislocations.
  - e.g C atoms in octahedral interstice of α-Fe
  - smaller size means rapid diffusion towards the dislocation. C atoms can ~~surround~~ form along the dislocation line : Cottrell atmospheres

low-carbon steel



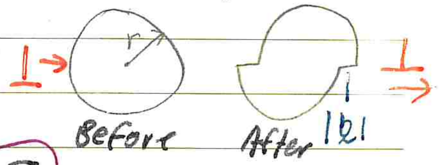
- Cottrell atmospheres provide strengthening because it is expensive to move them.
- Plastic deformation begins at the sample ends
  - no longer pinned by Cottrell atmospheres so continue to deform → but they also work harden, so the unyielded regions start to yield
  - the boundaries are Lüders bands, lying at ~50° to the tensile axis and moving towards each other.
  - when the bands meet, the entire sample work hardens as expected.
- This strengthening is highly temperature-dependent
  ↳ at high temp C atoms are very mobile and move with the $\perp$s so there is no strengthening effect until work hardening occurs.

- At intermediate temp, C and ⊥ move at comparable speeds.
  - ↳ dislocations can escape Cottrell atmospheres only to be re-pinned
  - ↳ this is the Portevin-Le Chatelier effect and leads to a serrated stress-strain curve.



Precipitate strengthening

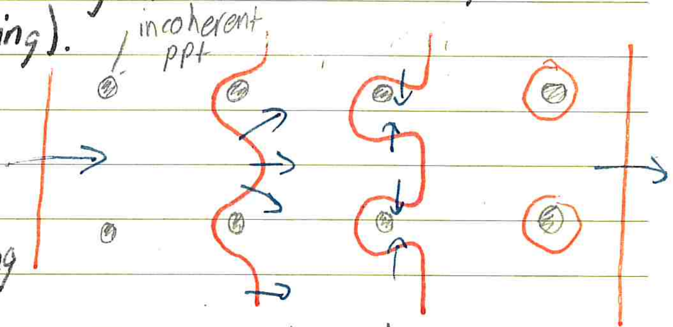
- Small precipitates tend to be coherent with the surrounding matrix → dislocations can pass through by precipitate cutting, but clearly this is harder than gliding through the matrix.
- Thus there is a strengthening effect: $\boxed{\Delta\tau \propto \sqrt{r}}$



- But beyond a certain radius, ppt is likely to be incoherent, so ⊥ cannot pass through (no cutting).
- Instead, they can pass around precipitates by Orowan bowing
- By considering the peak energy situation (semicircle) and equating the vertical force with the line tension from each dislocation:

$$\tau|\underline{b}|L = 2\left(\frac{G|\underline{b}|^2}{2}\right)$$

  - ↳ L is surface-surface separation

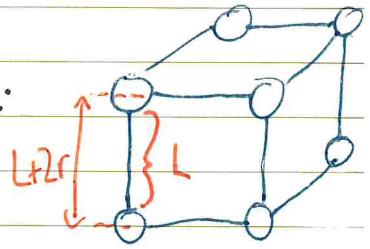


- Thus the Orowan stress required to bow a dislocation is

$$\boxed{\tau = \frac{G|\underline{b}|}{L}}$$

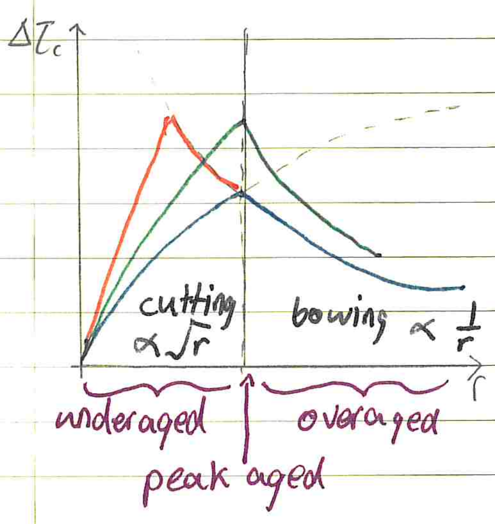
- Assuming the ppts have a cubic arrangement:

$$V_f = \frac{V_{ppt}}{V_{cube}} = \frac{\frac{4}{3}\pi r^3}{(L+2r)^3}$$



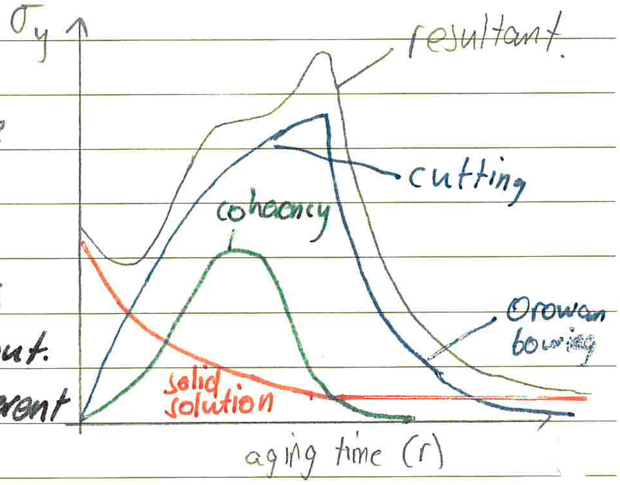
- So for a constant volume fraction, $L \propto r$
- Thus greatest τ (i.e strongest) when many small ppt particles.

- As a sample is aged, ppt radius will increase. There will be some ideal radius that has maximum resistance to cutting, before Orowan bowing becomes significant.



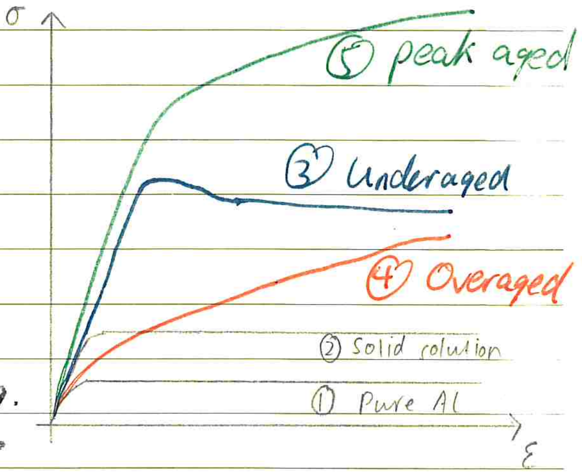
- With stronger precipitates:
  - ↑ cutting resistance
  - no effect on bowing
  - ∴ ↑ peak stress but at lower r
- With greater fraction of precipitates:
  - ↑ cutting resistance (more to cut)
  - ↑ bowing resistance because L↓
  - ∴ ↑ peak stress.

- But there are other factors that affect the yield strength.
- Initially after quenching, we will have a supersaturated solid solution
  - i.e solid solution strengthening from substitutional/interstitial solute atoms
  - this decreases as solute precipitates out.
- Initially, when ppt is coherent/semicoherent it distorts the matrix and inhibits dislocation motion – coherency strain
  ↳ but once ppt becomes incoherent, they become a dislocation source.



- For example, in the Al-Cu system:
  - ↑ Cu ↑ strength (greater ppt fraction)
  - ↓ temp ↑ peak stress, because the coherent phases have more time to separate and grow
  - but there is a tradeoff because ↓ temp requires much longer time to reach peak strength
  - the optimal strength sample will have both θ' and θ'' phases, with both cutting and bowing.

Precipitates and work hardening



① Pure Al has a low yield strength and will only have a little work hardening from $\perp$ interactions

② A solid solution alloy will have higher σ_y than pure Al but the same hardening.

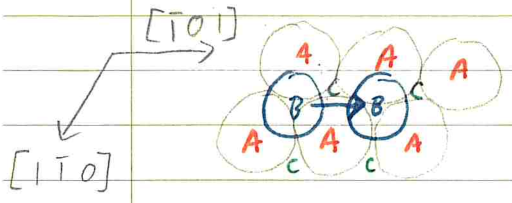
③ Underaged sample will contain GP zones that provide significant strengthening. But once slip starts, $\perp$ can pass through the entire crystal and shear the GP zones

④ Overaged has low σ_y because $\perp$s will bow around ppt. But bowing produces dislocation debris (loops and tangles) that inhibit further $\perp$ movement ⇒ rapid work hardening

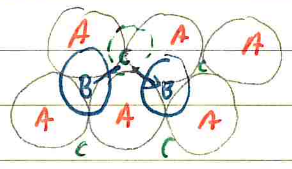
⑤ Peak aged material has the best of ③ and ④, having high σ_y due to cutting, and rapid work hardening from bowing.
↳ this is why the best alloys have θ'' and θ'.

Other deformation mechanisms

Partial dislocations and stacking faults



Perfect dislocation
B → B
$b_1 = \frac{a}{2}[\bar{1}01]$



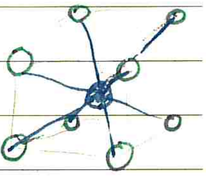
Partial dislocation
B → C: $b_2 = \frac{a}{6}[\bar{2}11]$
C → B: $b_3 = \frac{a}{6}[\bar{1}\bar{1}2]$

- Instead of a perfect dislocation, an alternative path involves two partial dislocations (which aren't lattice vectors).
- A perfect $\perp$ will dissociate if it is energetically favourable (Frank's rule). Partial $\perp$s are always mixed.
- Note that after passage of the first partial $\perp$, the stacking sequence has changed:

initial: A B C A ¦ B C A B C
after b_2: A B C (A ¦ C A) B C A
slip
stacking fault.

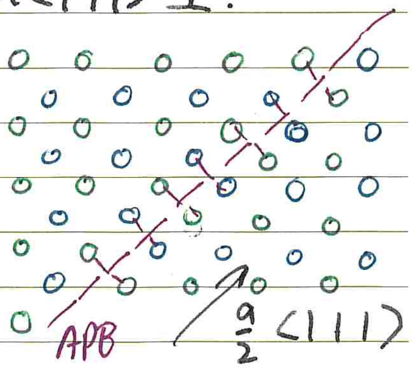
- when a perfect $\perp$ dissociates, the elastic stress fields of the partial $\perp$s repel, causing them to separate, causing an extended stacking fault region to appear
  - ↳ width determined by stacking fault energy (SFE). If SFE is high, partial $\perp$s will be close by.
- Partial $\perp$s cannot cross-slip because they have an edge component – they must recombine before they can change plane.
  - ↳ materials with low SFE (harder to recombine $\perp$s) will thus work harden rapidly.

Order hardening



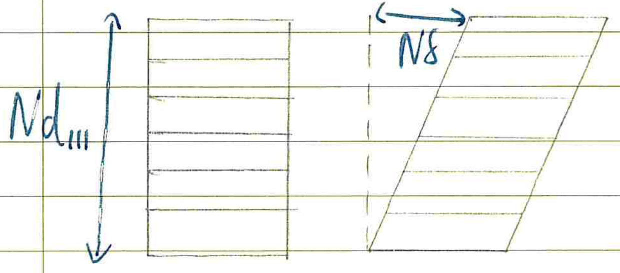
- At low temperatures, β brass has an ordered CsCl structure (cubes of Cu with Zn in centre).
  - ↳ we would expect slip with $b = \frac{a}{2}\langle 111 \rangle$ but this is not a lattice vector and would create expensive Cu-Cu bonds.
- But at high temp it is a random solid solution, so $b = \frac{a}{2}\langle 111 \rangle$ corresponds to a perfect $\perp$
  - ↳ thus there is a large drop in τ_c at a particular temp.

- But if we apply enough stress to low temp β brass it will still have to slip: it does this by passage of two $\frac{a}{2}\langle 111 \rangle$ $\perp$s to give a perfect $a\langle 111 \rangle$ $\perp$.
  - passage of $\frac{a}{2}\langle 111 \rangle$ creates an anti-phase boundary (APB), ~~&~~ an energetically unfavourable layer of like-like bonds.
  - as with stacking fault regions, the two partial $\perp$s will repel to form a superdislocation (which includes both partial $\perp$s and the APB).
  - thus order hardening provides significant strengthening, even at relatively high temp (as long as it's below the order→disorder temp).



Deformation by cooperative shear

- Cooperative shear is an alternative deformation mechanism that does not involve ⊥ motion.
- Instead, succesive planes are sheared, which can form the same crystal structure but at a different orientation
  - ↳ i.e deformation twinning
  - ↳ each plane is only sheared a small amount, with no changes in bonding.
- The twinning direction will be the same as the partial ⊥ direction, e.g $\frac{a}{6}\langle 1\,1\,\bar{2}\rangle$ for fcc.



- the twinning plane for fcc is (111).

$$d_{111} = \frac{a}{\sqrt{3}} \quad \text{and} \quad \delta = \frac{a\sqrt{6}}{6} \leftarrow |\underset{\sim}{b}| \text{ equivalent}$$

- ∴ maximum shear:

$$\gamma = \frac{\delta}{d_{111}} = \frac{1}{\sqrt{2}}$$

- Thus, twinning can facilitate significant deformation ($\frac{1}{\sqrt{2}} \approx 70\%$).

- Twinning is an athermal process, so may be the dominant deformation mechanism at cryogenic temperatures.
- Twinning is more likely for crystal systems with fewer slip systems, for which dislocations are less mobile.
- Twinning is more likely with low SFE, such that regions of a different stacking pattern (i.e twins) may form
- Deformation twinning can propagate rapidly (~ speed of sound), so may be the favoured mechanism at high strain rates.

- Deformation twins have a lenticular shape (such that they are compatible at grain boundaries), allowing them to be distinguished from annealing twins.
  - ↳ also, lens shape minimises strain field.

Fracture

- Fracture is the separation of a solid body under the influence of an applied load, involving the creation and propagation of cracks.
- Toughness is the resistance of a material to crack propagation.
- Ductile materials undergo plastic deformation before fracture, whereas it is much more sudden for brittle materials.
- We can estimate fracture stress by considering interatomic forces.

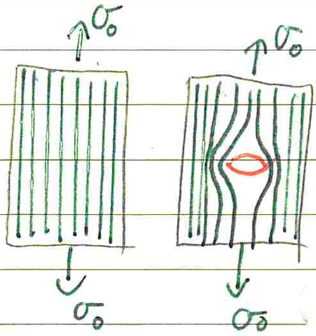
$$E = \frac{1}{r_0} \frac{d^2U}{dr^2}\bigg|_{r=r_0} = \frac{72U_0}{r_0^3}$$

$$\sigma = \frac{F}{A} = \frac{1}{r_0^2}\frac{dU}{dr} = \frac{E}{6}\left[\left(\frac{r_0}{r}\right)^7 - \left(\frac{r_0}{r}\right)^{13}\right] \Rightarrow \boxed{\sigma_{max} \sim 0.04E}$$

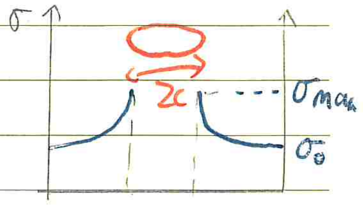
↳ but this model greatly overestimates fracture stress.

⇒ failure is controlled by defects in the material.

Cracks

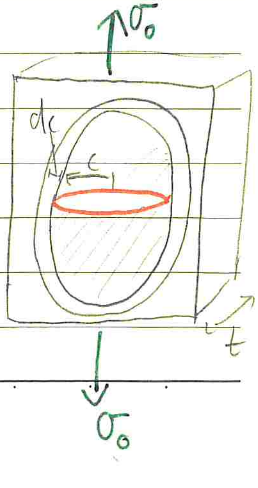


- For a defect-free material, stress will be transmitted evenly.
- But if there is a crack, the lines of force will have to go around it, creating a stress-free region directly above and below the crack, but also a stress conc. at the tips.



- σ_{max} depends on the crack length and the radius of curvature of the tip

$$\sigma_{max} = \sigma_0\left(1 + 2\sqrt{\frac{c}{r}}\right)$$



- When a crack grows, it releases stored elastic strain energy in the surrounding material.
  - if we model the oval as 2x circle of radius c, $dV = (2\pi c\,dc) \times 2 \times t$ ← thickness
  - elastic strain per unit volume is $\frac{\sigma_0^2}{2E}$

$$\therefore dW = \frac{\sigma_0^2}{2E} \cdot 2 \cdot (2\pi c t dc) = \frac{2\sigma_0^2 \pi c t}{E} dc$$

- The strain energy release rate G is the rate of release of strain energy per unit of new crack area (time-independent).

NOT SHEAR MODULUS

$$\therefore G = \frac{dW}{\text{new crack area}} = \frac{dW}{4 t dc} = \frac{\pi \sigma_0^2 c}{2E}$$

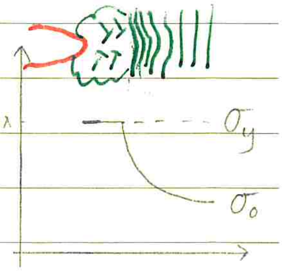
does not agree w/ experiment. Remove factor of 2 because we assumed constant σ_0

- The constant~~s~~ depends on geometry, but $\boxed{G \sim \frac{\sigma_0^2 c}{E}}$ is general.
  - ↳ ↑ G if stress ↑ or if crack length ↑
  - ↳ i.e longer cracks release more energy per unit area
- In order for a crack to propagate, G must exceed some critical value, which will be 2γ if γ is the surface energy of a brittle material
  - ↳ Griffith criterion: $G \geqslant G_c = 2\gamma$
  - ↳ a.k.a crack resistance or work of fracture.
- G_c can either be used to find the critical stress for a given crack length, or the critical flaw size for a given stress

$$\sigma^* = \sqrt{\frac{2\gamma E}{\pi c}} \quad \text{or} \quad c^* = \frac{2\gamma E}{\pi \sigma_0^2}$$

- The fracture stress of a material can be raised by polishing the surface or compressing it, such that cracks become shorter.
- Corrosive environments or fatigue can cause cracks to elongate.

Ductile fracture



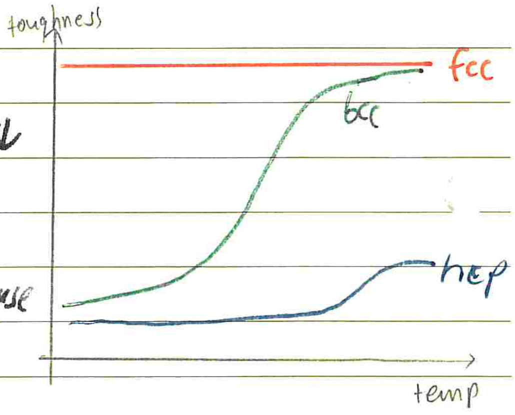
- In a ductile material, the stress conc. leads to plastic deformation occurs near the crack tip, blunting it and thus reducing stress.
- ∴ Work must be done for plastic deformation, so much higher strain rate required to propagate crack.
  - ↳ use modified Griffith criterion: $G \geqslant G_c = 2(\gamma + \gamma_p)$
  - ↳ $\gamma_p \gg \gamma$ (by a factor of ~1000).

Stress intensity factor

- Because it is very difficult to determine the strain energy release rate G, we introduce the stress intensity factor: $K \equiv \sigma_0\sqrt{\pi c}$
- When K reaches a critical value (the fracture toughness), fracture occurs. $K_c = \sqrt{G_c E}$ is a material property.
- There are non-destructive testing methods to evaluate K in materials, which can be compared to K_c during audits.

Ductile-brittle transition temperature

- Heating leads to increased plasticity and greater toughness.
- Likewise, cooling makes a sample more brittle.
- Increasing the strain rate also inhibits $\perp$ motion so is similar to cooling, but cooling has a much greater effect.
- The toughness of a material can be measured by impacting the sample with a pendulum. The rebound height tells us the impact energy - a proxy for toughness.
- fcc metals are tough at all temps because of its slip systems → mobile $\perp$s.
- bcc shows a large loss of toughness as T↓ because some slip systems require thermal activation and $\perp$s prone to pinning
- hcp is brittle even at ambient temp because only one slip system can activate.



Toughness of composites

- Composites can be tough even when both the matrix and fibres are brittle.
- This is because work must be done on the material to pull fractured fibres out of matrix sockets
- A crack in the matrix can only propagate once embedded fibres are out of their sockets - fibre pull out

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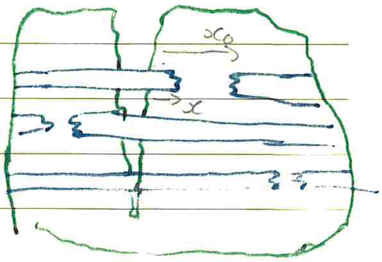


- Work is required for interfacial debonding, which is a 'friction' that depends on the interfacial shear stress τ_i^*:

$$dW = (2\pi r x)\, \tau_i^*\, dx$$

$$\Rightarrow \Delta W = \int_0^{x_0} (2\pi r x)\, \tau_i^*\, dx = \pi r x_0^2\, \tau_i^*$$

- The number of fibres per unit area for a volume fraction f is given by $M = f/\pi r^2$
- Hence the pull-out work of fracture (G_{cp}) is:

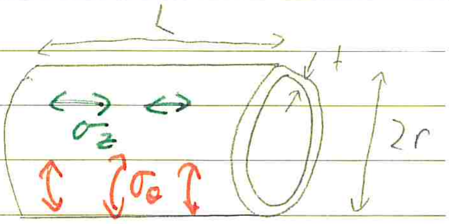
$$G_{cp} = M \Delta W = \frac{f \pi r x_0^2 \tau_i^*}{\pi r^2} = 4 f s^2 r \tau_i^*$$

where s is the pull-out aspect ratio $s \equiv x_0/2r$

↳ $s \sim 20\text{-}50$, i.e. 100μm long and 10μm thick.

- The fibres do not tend to break in the crack plane because they are likely to have defects elsewhere.

Pressurised pipes



- Three stress components:
  - axial σ_z along pipe
  - circumferential (hoop) σ_θ around pipe
  - radial σ_r across wall. $\sigma_r \approx 0$ for thin walls.
- The cross sectional area of the walls when cut along the pipe is $2tL$, so if the pressure is P and pipe in equilibrium:

$$\sigma_\theta (2tL) = P(2rL) \Rightarrow \sigma_\theta = \frac{rP}{t}$$

projected area

- Similarly for the axial case:

$$\sigma_z (2\pi r t) = P(\pi r^2) \Rightarrow \sigma_z = \frac{rP}{2t}$$

- Because hoop stress = 2x axial stress, pipes tend to burst longitudinally.

A'ZONE