

F. Biomaterials

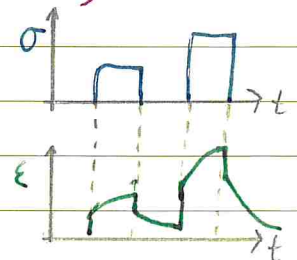
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- A biomaterial is one that comprises part of a living structure or a biomedical device that augments / replaces natural functioning.
- Biomaterials often have much smaller embodied energies and carbon footprints, as well as being recyclable.
- Natural materials are mainly made of C, H, O, N with small amounts of Ca, P, Fe.
- Biomaterials tend to be highly anisotropic because they are often composites.
- Only tend to work at ambient temperatures.

Materials selection

- When a stress is applied to a linear-elastic material, it instantly becomes strained. $\tau = G\gamma$
- However, polymers may have behaviour more similar to liquids, in which the flow rate depends on the stress
↳ for a Newtonian fluid, strain rate \propto stress
 $\therefore \tau = \eta \frac{d\gamma}{dt}$ where η is the viscosity (Pa.s)
- In reality, materials exist on a spectrum of viscoelasticity.
- Organic materials tend to have 3 strain components:
 1. Elastic
 2. Viscous flow
 3. Slow recovering



- These materials can be modeled as combinations of springs (linear elastic solids) and dashpots (ideal liquids). e.g.

$$\begin{aligned} \text{↳ } \epsilon &= \epsilon_d + \epsilon_s & \therefore \dot{\epsilon} &= \dot{\epsilon}_d + \dot{\epsilon}_s \\ \Rightarrow \dot{\epsilon} &= \frac{\sigma}{\eta} + \frac{\dot{\sigma}}{k} & \Rightarrow k \dot{\epsilon} &= \frac{1}{t_R} \sigma + \dot{\sigma} \end{aligned}$$

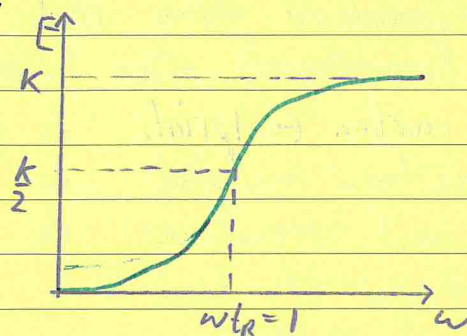
relaxation time
 $t_R = \frac{\eta}{k}$

(Maxwell model)

We can impose an oscillating stress $\sigma = \sigma_0 e^{i\omega t}$ which will cause a strain $\epsilon = \epsilon_0 e^{i(\omega t - \phi)}$:

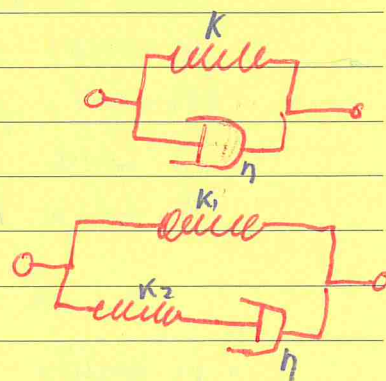
$$\Rightarrow E = \frac{K\omega^2 t_R^2}{1 + \omega^2 t_R^2}$$

$$\tan \phi = \frac{1}{\omega t_R}$$



- at high freq, there is no time for viscous flow so the material behaves like a linear elastic solid. $E \rightarrow K, \phi \rightarrow 0$
- at low freq, spring is negligible compared to dashpot. Behaves like an ideal liquid. $E \rightarrow 0, \phi \rightarrow \pi/2$
- at intermediate frequency, some strain will be recovered but energy will be dissipated.

- The Voigt model is more accurate for constant stress - some time dependence, but $\epsilon \rightarrow$ constant.
- However, it doesn't work well at high freq because E is dominated by the dashpot. The standard linear solid model improves this.

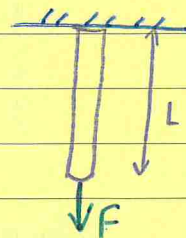


Selecting materials for strength

- Suppose we want to choose the lightest material for a wire of length L that must support a load F :
 - the materials properties to be considered are the failure strength σ_F and density ρ .
 - the thinnest wire that can be used has

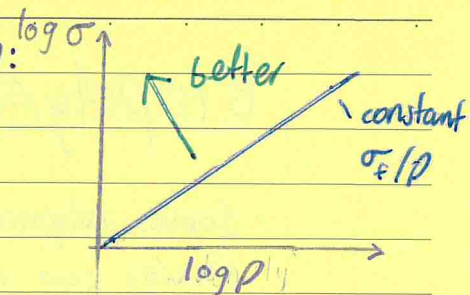
$$\sigma_F = \frac{F}{\pi r^2} \quad \text{and} \quad \rho = \frac{m}{\pi r^2 L} \quad \xrightarrow{\text{eliminate } r} \quad m = LF \left(\frac{\rho}{\sigma_F} \right) \quad \leftarrow \text{materials properties.}$$

- thus to minimise mass we maximise the merit index σ_F/ρ
- r was not specified in the problem so was eliminated as a free variable.



- We can then look at a strength-density map:

- contours of constant merit can be plotted
- in this setup, woods and steels are similarly 'good', but spider silk is the best.



- Consider a rectangular cantilever of fixed length & width. Suppose we want to minimise mass for a given load:

- at the top of the root $\sigma_f = \frac{Eh}{2R} = \frac{Mh}{2I} = \frac{6LF}{wh^2}$

- eliminating wh gives a merit index of $\sqrt{\sigma_f/\rho}$
- this has a gradient of 2 on the log-log scale.

Selecting for stiffness

- e.g. Minimise the deflection of a square cantilever with mass m .

$$\delta = \frac{FL^3}{3EI} = \frac{4FL^3}{Eh^4} \quad \text{and} \quad \rho = \frac{m}{Lh^2} \Rightarrow \delta \sim \frac{\rho^2}{E}$$

\therefore maximise the merit index $\sqrt{E/\rho}$

- Woods perform very well here.

Selecting for toughness

- To maximise resistance to an impact, we just need max G_c .

- To maximise the tensile load without crack propagation:

$$\sigma_f = \sqrt{\frac{EG_c}{\pi c}} \quad \therefore \text{maximise } \sqrt{EG_c}$$

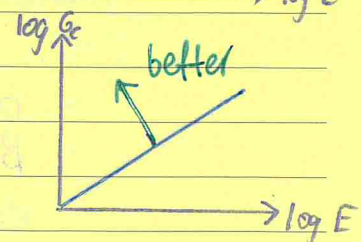
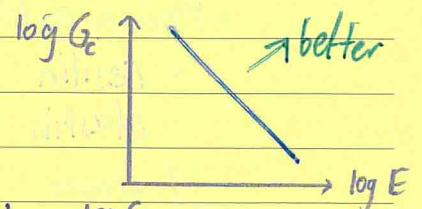
\hookrightarrow this is the fracture toughness K_{Ic}

\hookrightarrow good materials include bone or antler

- To maximise tensile strain without crack propagation:

$$\epsilon_f = \frac{\sigma_f}{E} = \sqrt{\frac{G_c}{\pi c E}} \quad \therefore \text{max } \sqrt{\frac{G_c}{E}}$$

\hookrightarrow best material for this is skin.

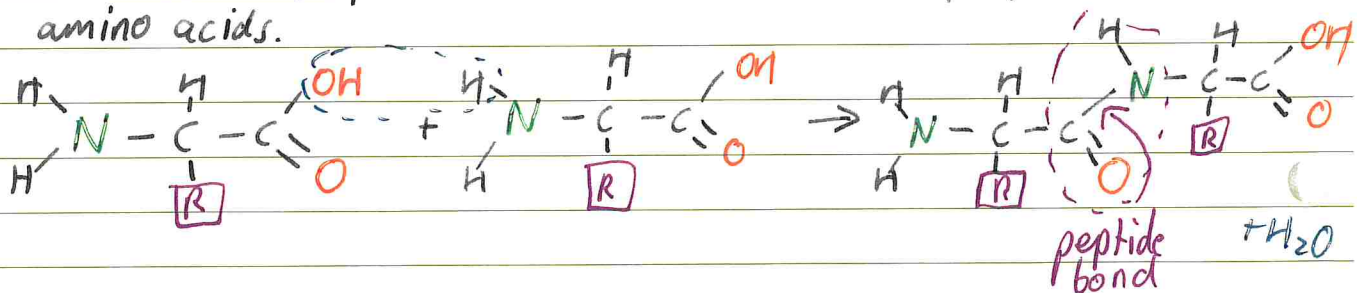


Biopolymers

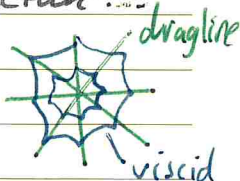

- Some polymers can show large recoverable strain because coiled chains can uncoil then return to lower energy conformation.
 - ↳ must be sufficient energy for bond rotation
 - ↳ chains must not slide past neighbours
- Natural rubber can be improved by **vulcanisation** to add sulfur crosslinks.
- Elastic behaviour depends on coiling rather than changing bond lengths: "**entropy spring**".
 - ↳ at high temp, rubber becomes stiffer again because $G = H - TS$, so ~~increasing~~ decreasing S by stretching the polymer results in $G \uparrow$.

Proteins

- Proteins are polymers made from condensation polymerisation of amino acids.

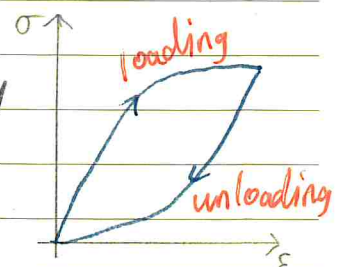
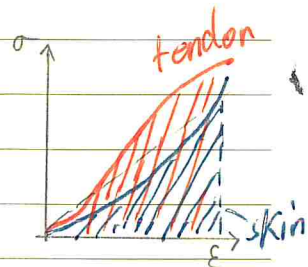


- Protein rubbers are randomly coiled cross-linked chains of proteins.
 - **resilin** has excellent elastic properties, found in insect wing hinges
 - **elastin** is found in vertebrates, e.g. human skin / neck ligaments.
- However, because of the hydrogen bonding between N-H and C=O, proteins can fold into compact conformations:
 - **α -helix**: right-handed helix with pitch $\sim 5 \text{ \AA}$
 - **β -sheet**, i.e. straightened chains lying in a plane.
 - very easy to transition between α and β because the H bonds can be broken by $T \uparrow$ temp or making it wet.

- Keratin is an α packed protein when dry, but β when wet.
 - hence wet hair can be shaped
 - chains already stretched and heavily crosslinked. Thus, keratin is strong and stiff.
 - Spiders can produce dragline silk and viscid silk (among others)
 - both have the same chemistry but different microstructure.
 - dragline silk is 25% β -sheet micelles: very high tensile strength / mass, used to support spider weight.
 - viscid silk is less crystalline and hence less stiff, but has a low coefficient of restitution - useful to catch insects.
- 
- Collagen is a protein fibre found in all multicellular animals
 - a collagen 'molecule' is made of three left-handed polypeptide helices coiled in a right-handed sense
 - these molecules arrange in a staggered pattern with covalent crosslinks to form fibrils that are stiff and strong.
- 
- fibrils are packed into fibres, in parallel bunches
 - Skin is a composite made of collagen fibres in an elastin matrix
 - fibres may have a preferential orientation: skin is anisotropic
 - different types of skin have different proportions of collagen/elastin.

Elastic energy storage

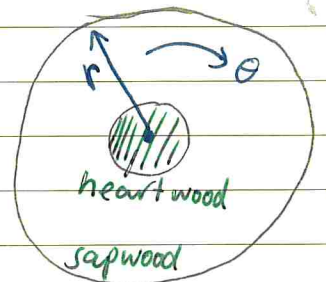
- Biomaterials rarely have a linear stress-strain curve
- Skin gets stiffer when stretched. This means that less energy is stored so crack propagation is harder.
- But materials that need to store more energy may have more convex curves, e.g tendon
- Because biomaterials are viscoelastic, input elastic energy is not completely recovered: elastic hysteresis
 - ↳ more area in loop \Rightarrow more energy absorbed during deformation, i.e tougher.



- Modeling a material as linear elastic, a merit index for energy storage is σ_F^2/E
- However, if we are interested in stored energy that can be returned, we must include the coefficient of restitution R (a.k.a. 'resilience').

Structural polysaccharides

- Wood is mostly made of vertically aligned elongated cells, with some additional radial cells.
 - ↳ highly anisotropic
 - ↳ can be analysed in cross section, radial section, or tangential section $\begin{matrix} \uparrow \\ \rightarrow \end{matrix}$



- In **softwood**, vertical cells are for support and conduction, with radial cells used for storage.
- In **hardwood**, vertical cells only support - there are specialised cells (**vessels**) for conduction.



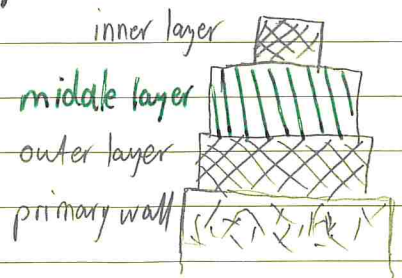
soft



hard

- The cell walls are a composite consisting of cellulose fibres in a lignin matrix. The insides are essentially voids.
- Cellulose is a polymer chain built from glucose units
 - chains are packed into small bunches
 - bunches stack together to form crystalline **microfibrils**
 - these microfibrils are separated by amorphous lignin, which makes up 50% of the material.

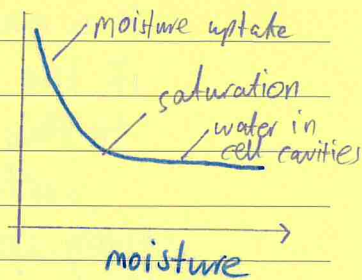
- The cell wall has multiple layers, the thickest being the middle layer which contains near-vertical fibres and is the main contributor to stiffness.



- Tensile stiffness can be modeled with the Voigt model - but the 'voids' should be included.

- Wood is much weaker in compression because of its fibres. Cell walls can buckle, causing **creases** which can become cracks in tension.
 - ↳ trees solve this by putting outer layers in tension, so much more compression can be tolerated.

- Wood's strength is strongly affected by moisture: σ_f
 - ↳ water helps break the H bonds between cellulose chains, which become more mobile.
 - ↳ varying moisture content can cause dimensional change.



- Wood is very tough; in addition to fibre pullout, separation of the middle and outer layers is very energy intensive.

Chitin

- Chitin is similar to cellulose: nitrogen-containing polysaccharide
 - ↳ stronger H bonds \therefore chitin is stronger and stiffer than cellulose.
- Chitin is the main structural material in exoskeletons, but it achieves its strength and hardness without incorporating minerals.
- Toughness similar to cortical bone.

Biominerals

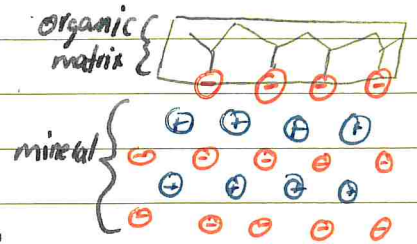
Non-structural biominerals

- Some bacteria are **magnetotactic**: they contain ferromagnetic crystals whose size, shape, and orientation are well controlled by proteins.
 - ↳ this allows the bacteria to orient themselves
- The inner ear contains small calcite crystals which move around within fluid and interact with small hairs - used for balance.

CaCO₃ for structural materials

- CaCO₃ is the main component of seashells, though a significant amount of Ca may be replaced by Mg.
- The main solid phases are: amorphous, calcite, aragonite.
 - ↳ though calcite is more energetically stable, especially when there are Mg²⁺ ions, both can be made to precipitate.

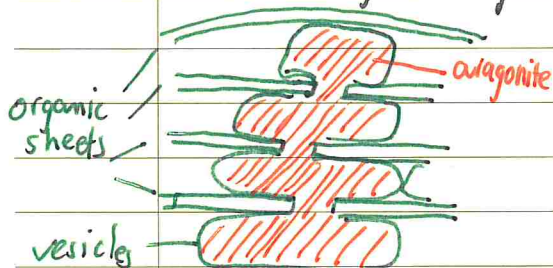
- It is easiest to nucleate a mineral structure if the organic template is well-matched
 ↳ thus living systems can encourage certain minerals to precipitate.



- Living systems also control ion transport and can thus specify the chemical composition.
- Constraints in growth geometry can lead to crystallographic texture (preferred directions), like in metal casting.

Nacre (mother of pearl)

- Nacre is a composite of organic protein sheets and aragonite.
- The aragonite grows to fill predefined vesicles in the organic matrix.

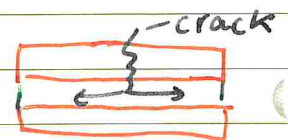


↳ this growth results in either a 'brick wall' or 'stack of coins' structure

- Though aragonite is very brittle, nacre is tough:
 - G_c 1000x greater
 - K_{Ic} 10x greater (because E is lower in nacre).

- Nacre's toughness is a result of crack deflection:

- when an aragonite plate is cracked, the crack will then deflect along the weak organic layers
 ↳ to plane of advancing crack



- this creates large new surface areas, absorbing energy.

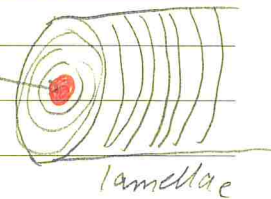
Biomedical materials

Bone

- Bone is a composite of collagen (soft) and **hydroxyapatite**, a hard mineral based on calcium phosphate.
- ↳ collagen fibres are interleaved with HA crystals to form a lamellar structure

↳ lamellar sheets wrap around the **Haversian canal**, a blood vessel, to form an **osteon** →

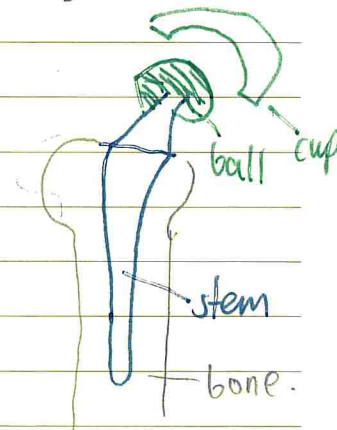
↳ these osteons are then bundled together to form the walls of **cortical bone**.



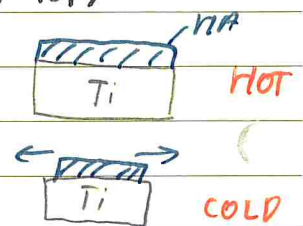
- Bone has reasonable tensile strength and stiffness (longitudinally), and is very tough — much tougher than artificial ceramics.
- **Cancellous / spongy bone** is less stiff: found at the ends of bones, its stiffness matches the cartilage layer so that stresses are shared.
- The body produces bone where it is needed
 - ↳ conversely, if bones are not subjected to stress, they are **resorbed**. Problematic for astronauts.

Artificial hip joints

- Metals are generally chosen for the stem:
 - ceramics are too brittle; polymers suffer from fatigue
 - blood is quite corrosive → need to choose metal that is biocompatible (nontoxic) and inert
 - e.g. titanium, stainless steel.
- Material cannot be too stiff, otherwise it will bear most of the load and bone may resorb
 - ↳ we use a porous Ti alloy, made by sintering, which is less stiff.

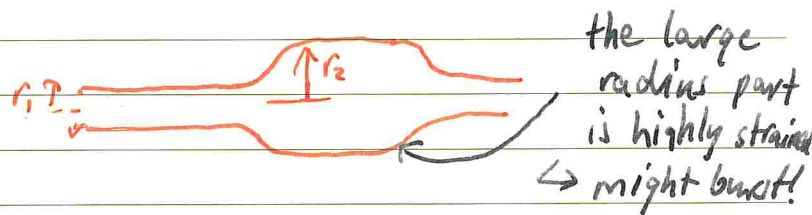
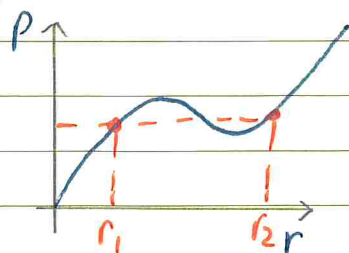


- The ball and cup require low friction and resistance to wear.
 - metal on metal has low wear but debris are metallic and may be a health risk.
 - current research involves ceramics, but they wear more easily.
- The stem is bonded to the bone by coating it with hydroxyapatite.
 - A problem is that the HA coating cracks off the metal:
 - HA coating is applied by **plasma spraying** (i.e. very hot)
 - because HA has a greater coeff of thermal expansion, it will be under tensile stress when cooled to room temp
 - this can be improved by alloying the Ti with manganese, which has higher α .

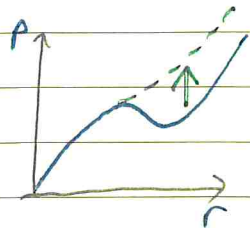


Arteries

- Arterial walls are a composite of collagen and elastin
- One problem is aneurysms:
 - the pressure ~~area~~ in an artery can be modeled as $P = \frac{\sigma t}{r}$, where σ is the hoop stress.
 - but because σ is a function of ~~extent~~ strain, one pressure can correspond to two stable radii



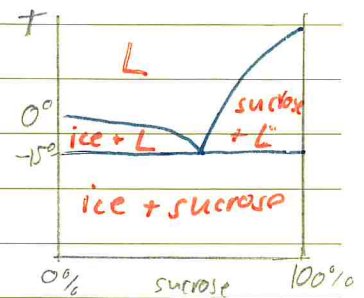
- to improve on this, we need a flexible material with a J-shaped stress/strain curve such that stiffness increases significantly on loading



Phase transformations in living systems

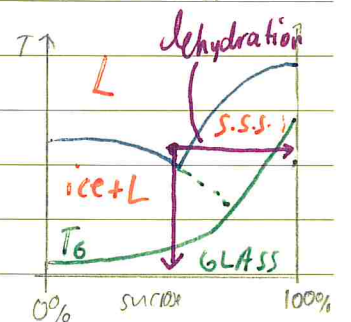
- Animal cells are made of a semi-permeable cell membrane
 - ↳ contains a liquid called **cytosol**, which can be modeled as a mixture of sugar and water.
 - ↳ formation of crystals in the cytosol is almost always fatal.

- The eq. phase diagram for sucrose-water is eutectic, but there is no solid solubility
 - ↳ i.e. ice crystals will be pure.



- We can change the phase by changing the temp. or by hydrating/dehydrating.

- Because crystallization of sucrose is kinetically difficult, the liquid can be supercooled a lot
- If it is supercooled enough, its viscosity may increase to the point where it can be considered a **glass**:

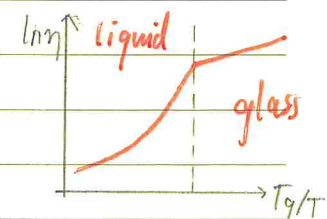


- cutoff is arbitrarily set at $\eta > 10^{12}$ Pa.s.

- the **glass transition** does not involve structural change or latent heat

- transition can result from cooling OR dehydration.

- viscosity as a function of temp. is non-Arrhenius



Dehydration in living systems

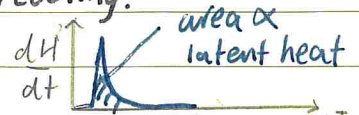
- When cells dehydrate, their cytosol becomes supersaturated and may crystallise.
- To combat this, some organisms ensure that there are high Mr sugars in the cytosol - more likely to form glasses than crystals
 - ↳ glasses preserve the cell
- This survival mechanism is found in desert plants, and is practically applied to dried foods such as pasta.

Freezing in living systems

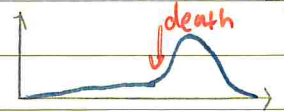
- Because the ice that forms is pure, the cytosol becomes more conc.
↳ cell draws in more water by osmosis and may rupture.

- The goal of living systems is to allow for supercooling.

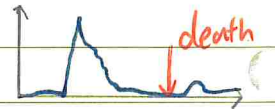
- bulk water can only be supercooled slightly before freezing.



- in hickory wood, freezing occurs outside the cells first (non-fatal).



- in peach flower buds, death only occurs when the last 5% of water freezes



- Freeze avoidance based on subdivision does not work for blood
↳ **antifreeze proteins** bind to growth sites on ice crystals and can thus prevent crystallisation for < 2 K supercooling.

- Some living systems encourage ice formation outside of cells
↳ Latent heat released protects the cells, and water diffuses out so the cytosol may form a glass (e.g. Northern Wood frog).

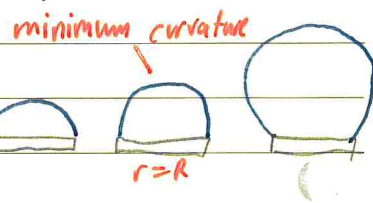
- To do this, intercellular regions contain **ice-nucleating agents (INAs)**.

- model the INA as a flat disc of radius R .

- as cap grows, radius of curvature \downarrow to min at R , then \uparrow

- thus for an ice crystal to grow, $r^* \leq R$

$$r^* = -\frac{2\sigma}{\Delta S \Delta T} \Rightarrow R = -\frac{2\sigma}{\Delta S \Delta T_{crit}} \Rightarrow \Delta T_{crit} = -\frac{2\sigma}{\Delta S R}$$



- Thus to catalyse nucleation for small supercooling, INAs should be as large as possible

- they are made of proteins that form large flat patches
- also have a hexagonal pattern with spacings that match the $\{0001\}$ planes on ice.