Kelly Betting

Q: What fraction $f^*$ of our bankroll should we allocate to a bet with a probability $p$ of winning $\$W$ and probability $1-p$ of losing $\$L$?

After $n$ such bets, our portfolio value $V_n$ is given by:

$$V_n = V_0 \left( \frac{1+fW}{1-fL} \right)^{pn} \left( \frac{1-fL}{1-fL} \right)^{(1-p)n}$$

We choose $f$ to maximise the log return:

$$\ln V_n = \ln V_0 + pn \ln(1+fW) + (1-p)n \ln(1-fL)$$

$$\frac{\partial \ln V_n}{\partial f} = 0 \Rightarrow \frac{\rho W}{1+fW} + \frac{(1-p)n(-L)}{1-fL} = 0$$

$$\therefore \frac{\rho W}{1+fW} = \frac{(1-p)L}{1-fL} \Rightarrow fW(1-p)L + fLPW = \rho W - (1-p)L$$

$$\Rightarrow f^* = \frac{\rho W - (1-p)L}{WL}$$